Neutron Star Simulations in Numerical Relativity

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Overview

- Relativistic hydrodynamics: equations of motion for ideal fluids

- Shocks and how to deal with them
  - Artificial viscosity
  - High resolution shock capturing schemes
- Applications
  - Collapse of rotating stars
  - Evolution and coalescence of binary neutron stars
  - Tidal disruption of neutron star by black hole companion
- Beyond ideal fluids
  - Viscosity
  - Magnetohydrodynamics

Relativistic Hydrodynamics

For ideal fluid

$$T^{ab} = \rho_0 h u^a u^b + P g^{ab}$$

where

$$h = 1 + \epsilon + P/\rho_0$$

Conservation of energy-momentum

$$\boxed{\nabla_b T^{ab} = 0}$$

Conservation of baryons

$$\boxed{\nabla_a(\rho_0 u^a) = 0}$$

Also need equation of state (EOS). Often use “gamma-law” equation of state

$$P = (\Gamma - 1)\rho_0 \epsilon$$

For isentropic flow this equivalent to polytropic EOS

$$P = K\rho_0^\Gamma \qquad\qquad \Gamma = 1 + 1/n$$

Wilson

Let

$$W \equiv -n_a u^a = \alpha u^t$$

be Lorentz factor between normal and fluid observers and define

$$D \equiv \rho_0 W \qquad E \equiv \rho_0 \epsilon W \qquad S_a \equiv \rho h W u_a$$

Then equations of motion become continuity equation

$$\partial_t(\gamma^{1/2} D) + \partial_j(\gamma^{1/2} D v^j) = 0$$

energy equation

$$\partial_t(\gamma^{1/2} E) + \partial_j(\gamma^{1/2} E v^j) = -P\left(\partial_t(\gamma^{1/2} W) + \partial_i(\gamma^{1/2} W v^i)\right)$$

and relativistic Euler equation

$$\partial_t(\gamma^{1/2} S_i) + \partial_j(\gamma^{1/2} S_i v^j) = -\alpha\gamma^{1/2}\left(\partial_i P + \frac{S_a S_b}{2\alpha S^t}\partial_i g^{ab}\right)$$

Have to be solved together with Einstein's equations

[Wilson, 1972; Hawley, Smarr & Wilson, 1984]

Numerical Implementations

Can adopt several different numerical techniques:

- finite differencing
- smoothed particle hydrodynamics (SPH)
- spectral methods

But...

Shocks

Generic initial data lead to formation of shock discontinuities

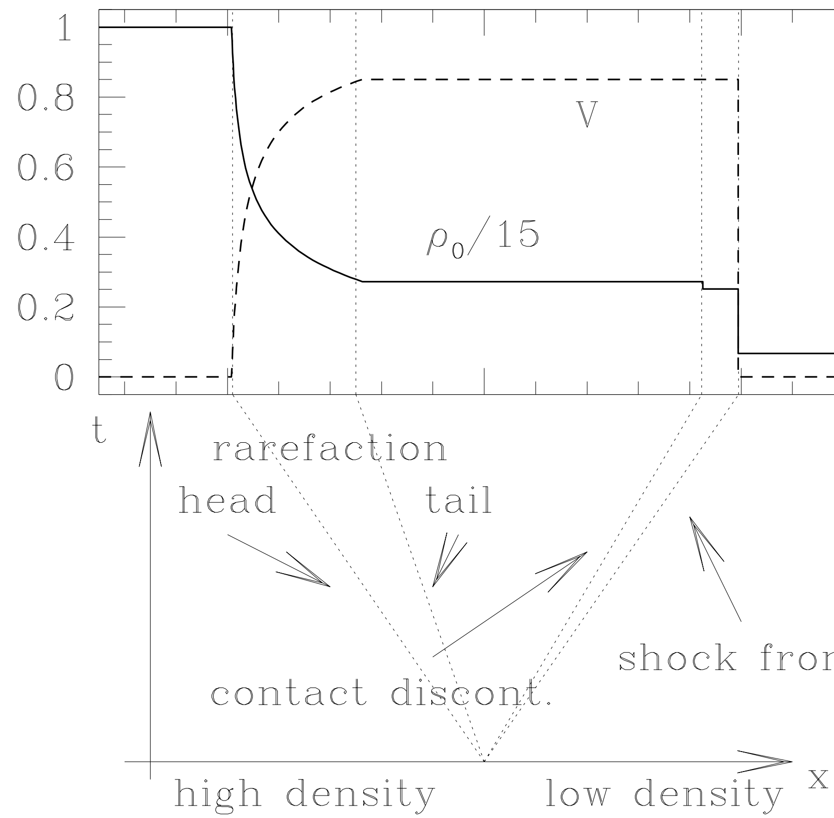
- E. g. relativistic Riemann problem - solution known analytically [Martí & Müller, 1991]

- fluid satisfies Rankine-Hugoniot jump conditions

- conversion of macroscopic kinetic energy into microscopic kinetic energy: heat

- straight-forward numerical implementations cannot mimick this process

$\Longrightarrow$ need additional feature



Artificial Viscosity

• Add artificial viscosity term $P_{\rm vis}$ to pressure when fluid is compressed

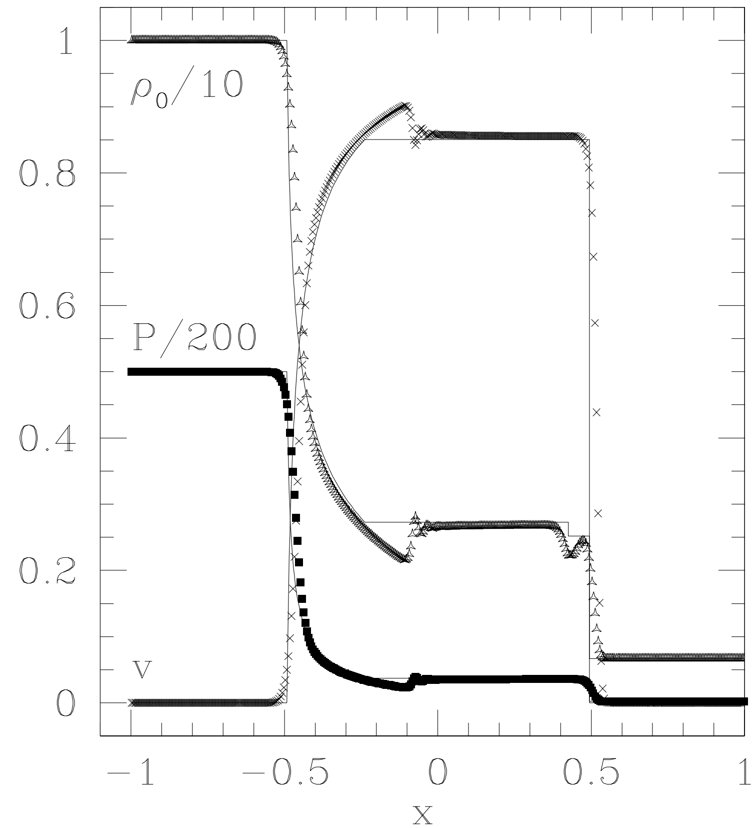
$$P_{\rm vis} = \begin{cases} C_{\rm vis}\rho_0(\delta v)^2 & \text{for } \delta v < 0 \\ 0 & \text{otherwise} \end{cases}$$

[von Neuman & Richtmyer, 1950]

• easy to implement

• works well for Newtonian fluids, less so for strongly relativistic shocks

• adequate in the absence of strong shocks

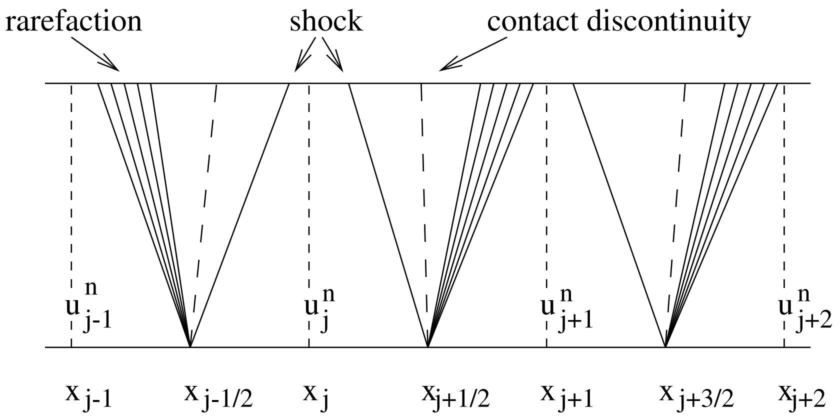


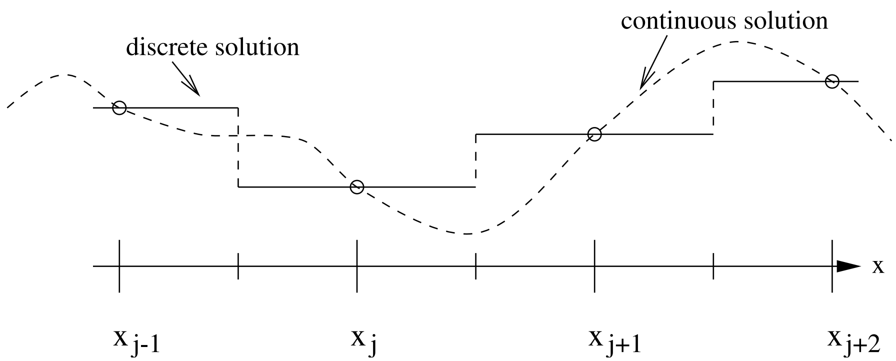
High Resolution Shock Capturing (HRSC) schemes

- Write equations in conservation form

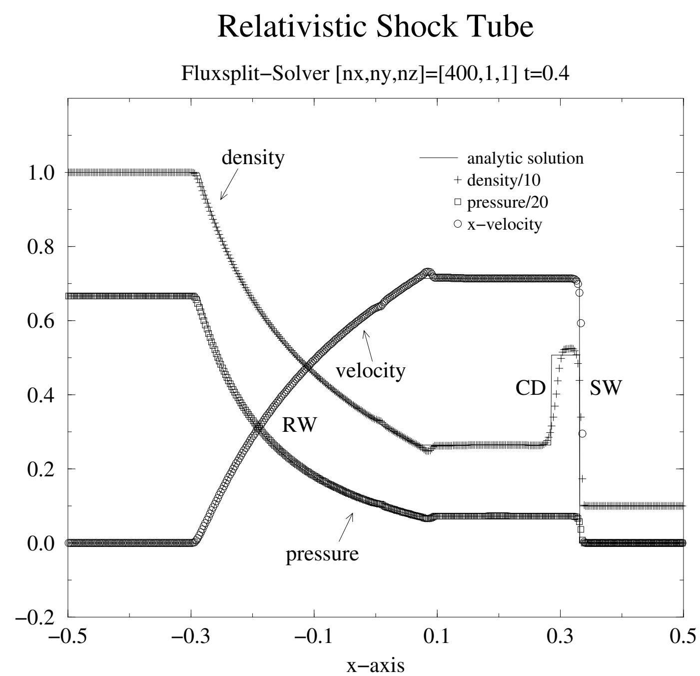
$$\partial_t \mathcal{U} + \partial_i \mathcal{F}^i = \mathcal{S}$$

- solve Riemann problem (approximately) at each grid interface [Godunov, 1959]





[Font, 2000, 2003]



[Font *et.al.*, 2000]

Existing Codes

- All the ones that I forgot...

- Finite differencing
  - Shibata *et.al.*, Illinois, Valencia/AEI/SISSA/WashU/LSU
  - use HRSC techniques
  - use BSSN formulation for gravitational fields
- SPH
  - Oechslin *et.al.*, Faber *et.al.*
  - use artificial viscosity
  - use conformal flatness approximation
- spectral methods
  - Meudon group
  - difficult to deal with discontinuities
  - only used in spherical symmetry?

Applications

- Collapse of rotating stars

- Evolution and coalescence of binary neutron stars

- Tidal disruption of neutron stars in black hole-neutron star binaries

Collapse of Rotating Stars

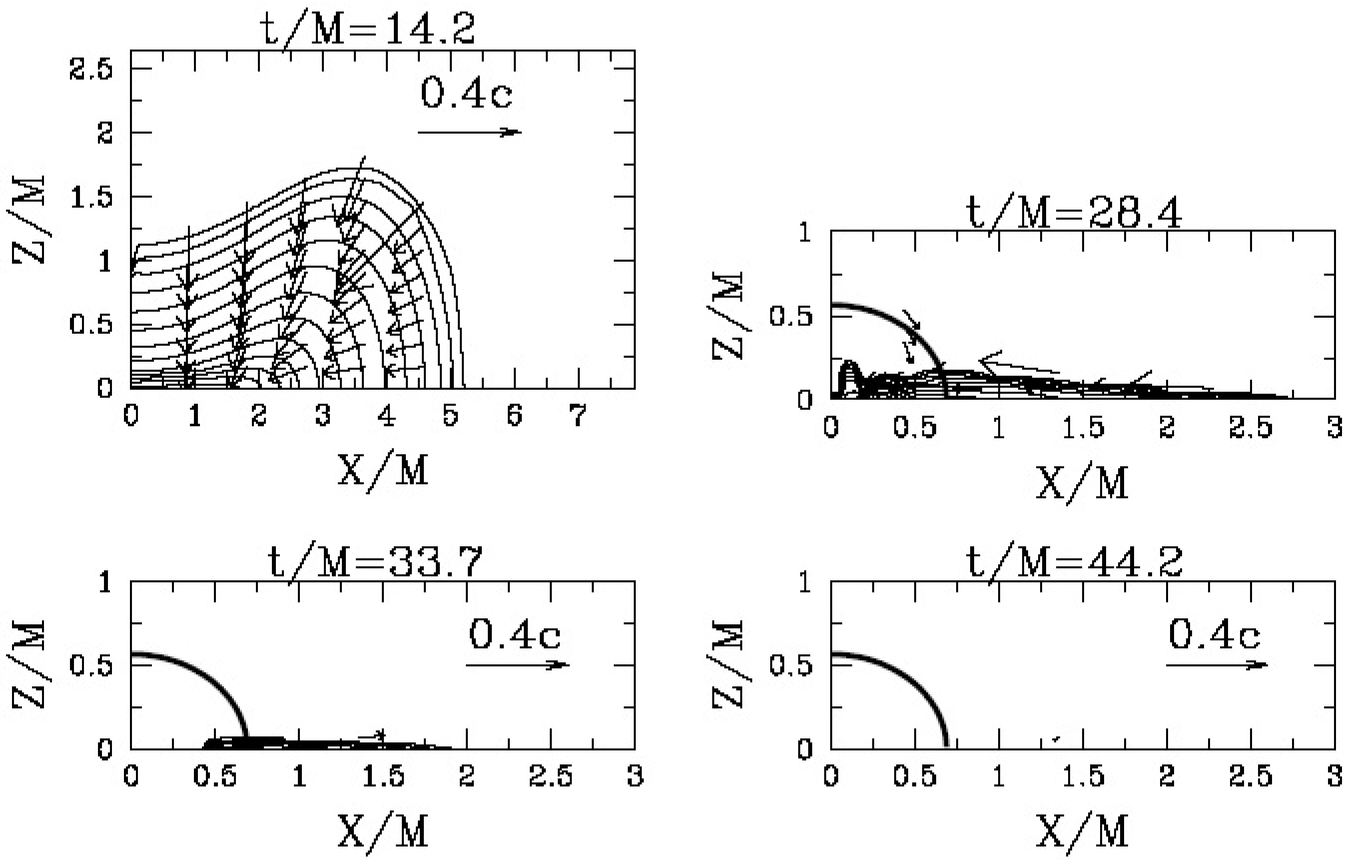
- Study collapse of rotating equilibrium polytropes to black hole [Shibata, 2003; Duez *et.al.*, 2004; Baiotti *et.al.*, 2005]
  - can use 3D code or 2D "cartoon" method [Alcubierre *et.al.*, 2001]
  - use excision for gravitational fields and fluids
  - induce collapse by pressure depletion

- Can study various properties of collapse, including difference between "subKerr" (with $J/M^2 < 1$) and "supraKerr" ($J/M^2 > 1$) collapse [Duez *et.al.*, 2004]

Collapse of "subKerr" Star

Differentially rotating polytrope with $J/M^2 = 0.91$

$\Longrightarrow$ black hole formation

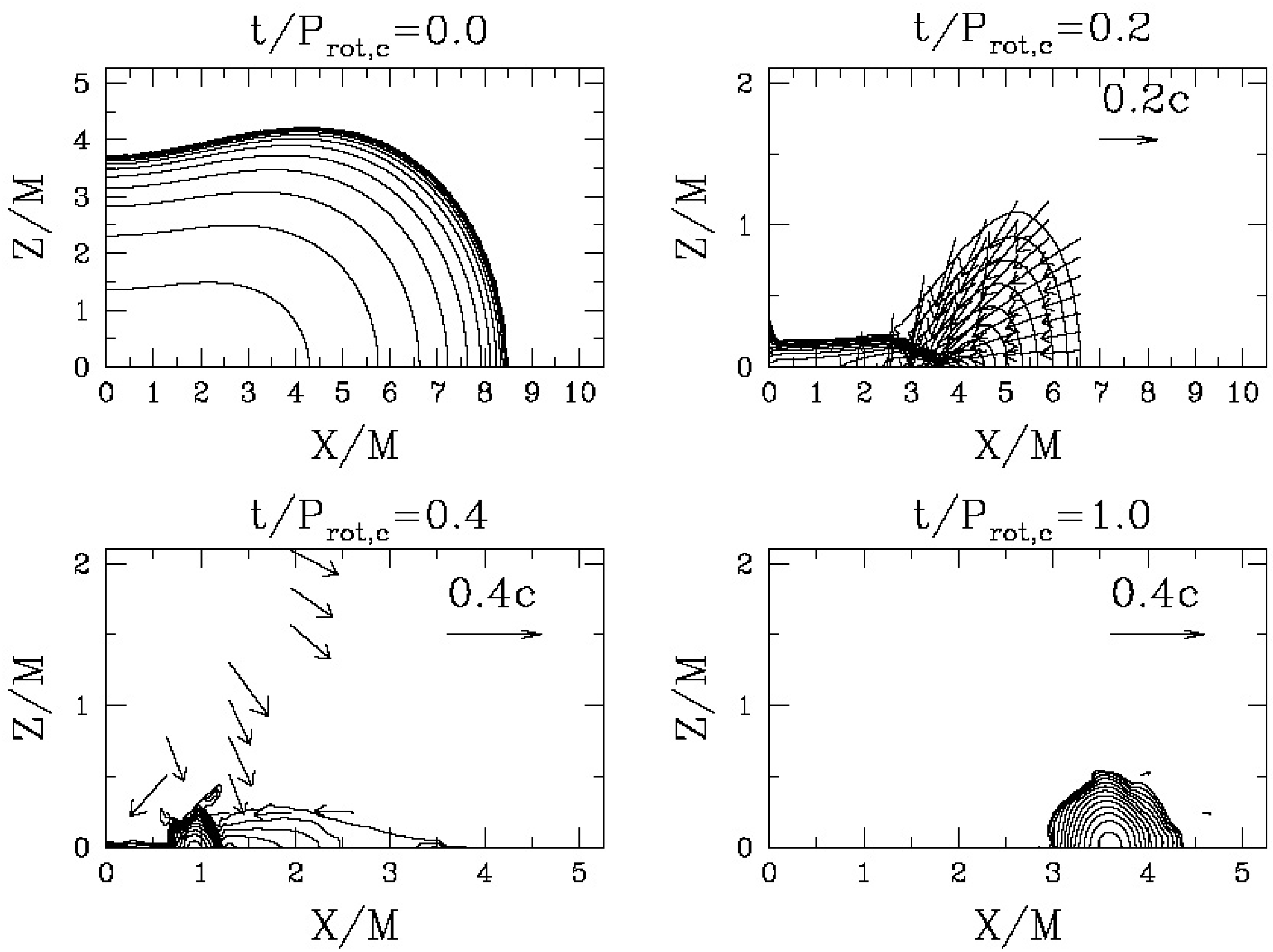


[Duez *et.al.*, 2004]

Collapse of “supraKerr” Star

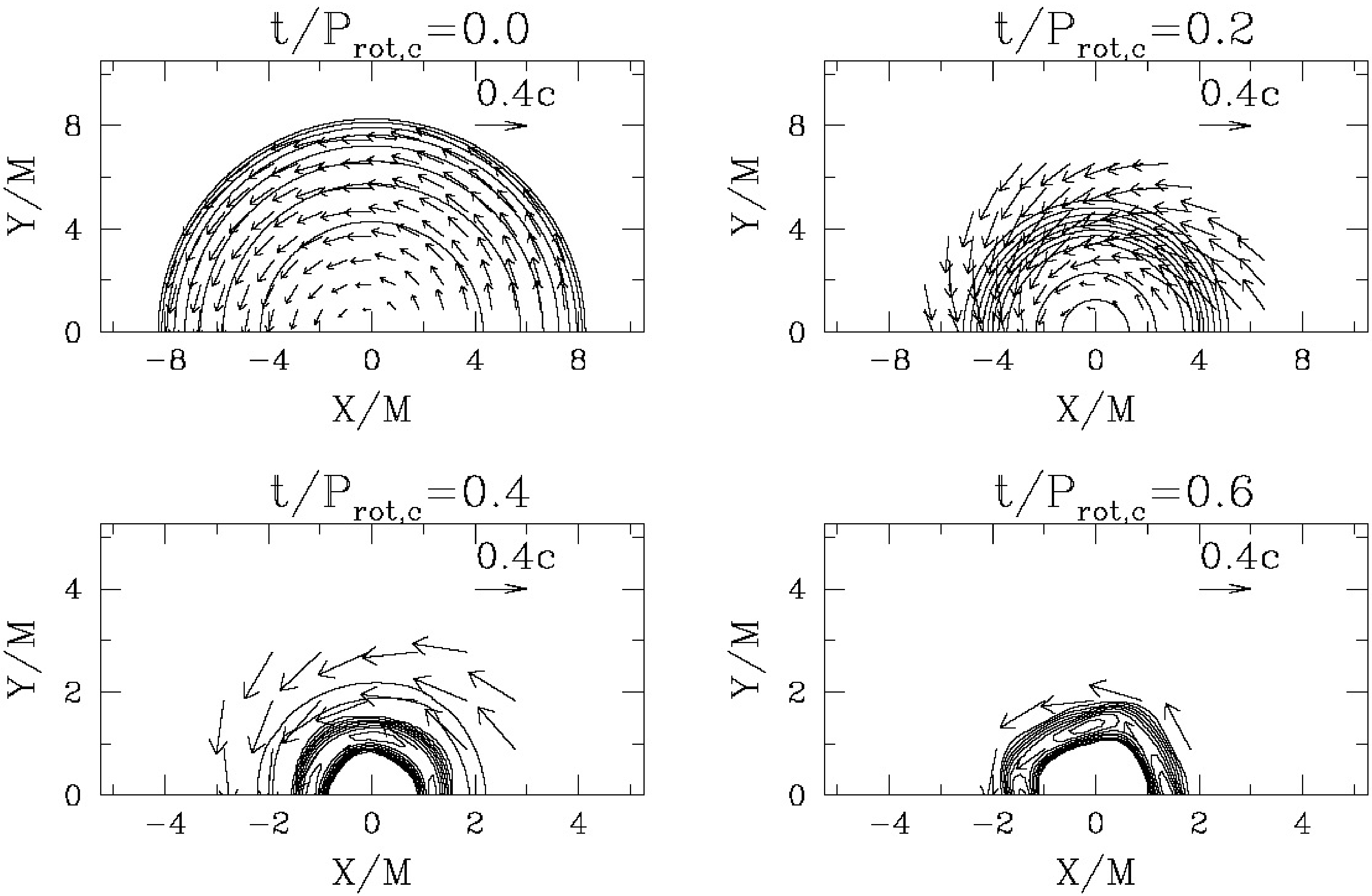
Differentially rotating polytrope with $J/M^2 = 1.18$

$\Longrightarrow$ no black hole formation



[Duez *et.al.*, 2004]

Fragmentation of torus



[Duez *et.al.*, 2004]

Evolution of binary neutron stars

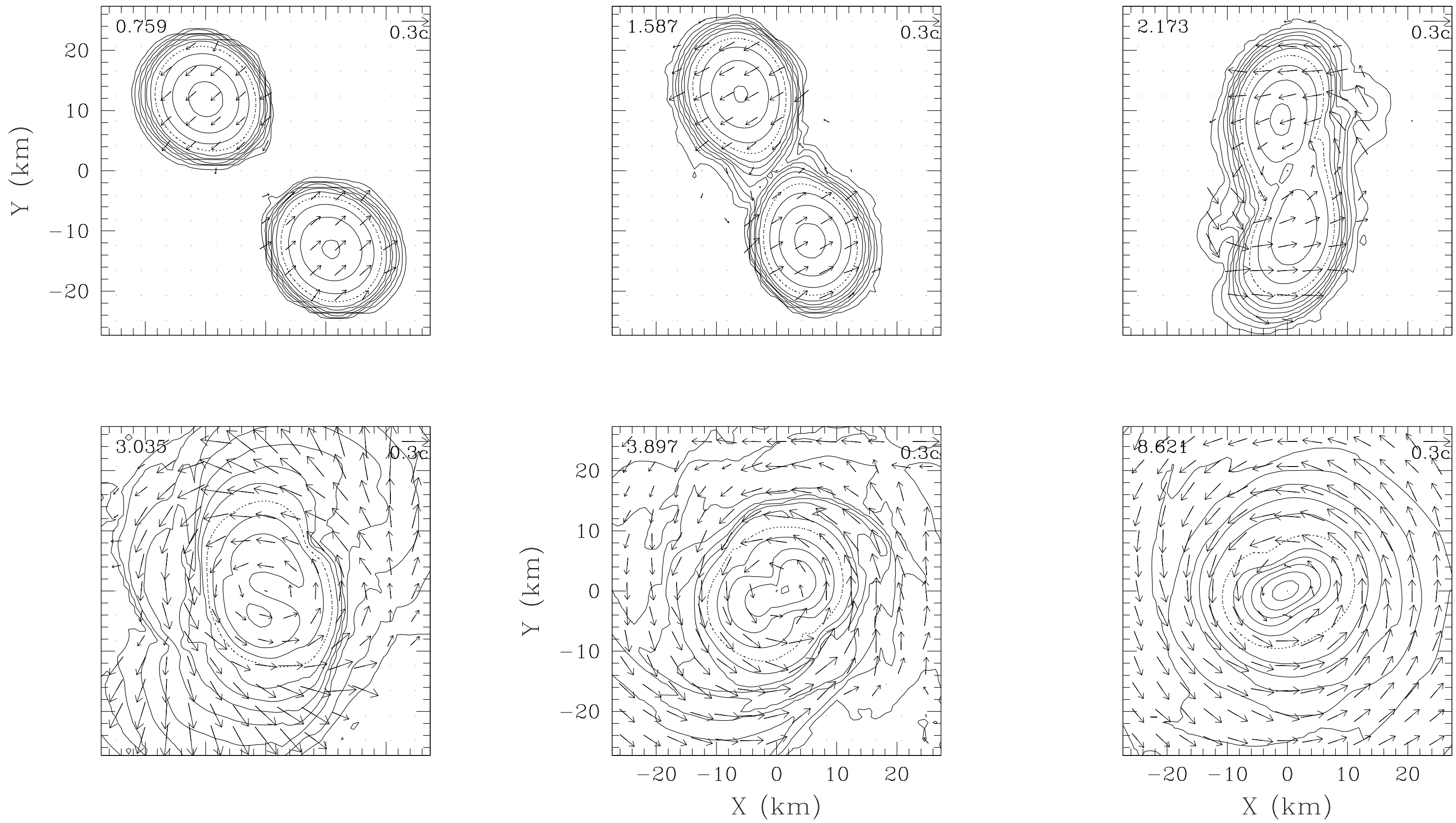
- Initial data: corotating or irrotational binary models in quasi-equilibrium

- Evolve binaries at varying separations to distinguish stable from unstable orbits
  $\Longrightarrow$ dynamically locate "ISCO"
  [Marronetti *et.al.*, 2004]

- Study binary coalescence
  [Shibata, 2000 ...; Oechslin *et.al.*, 2002; Faber *et.al.*, 2004]
  ◦ latest update: nuclear EOS instead of polytrope [Shibata *et.al.*, 2005]

Coalescence of binary neutron stars

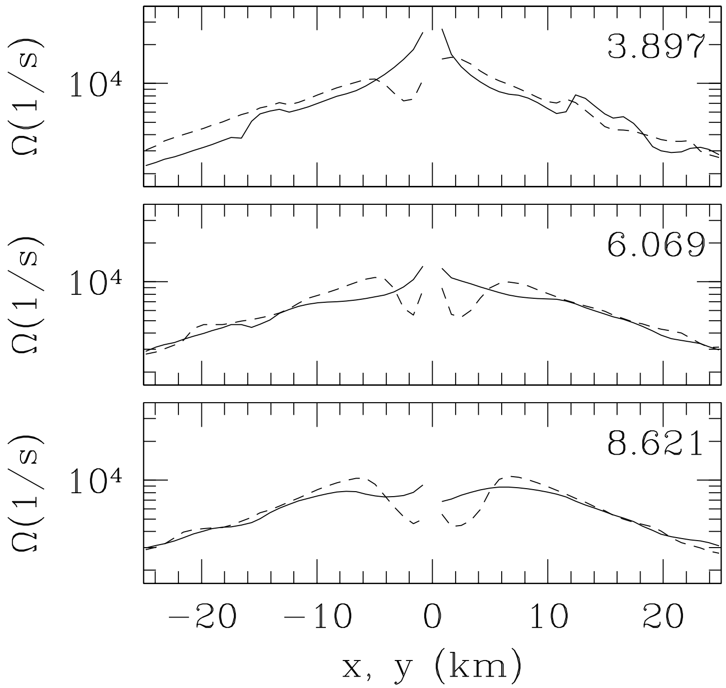
Example: irrotational binary, SLy EOS, rest masses of 1.25 and 1.35 $M_\odot$, sum exceeds maximum allowed rest mass by about 20 % [Shibata *et.al.*, 2005]



Hypermassive neutron stars

- Surprising result: remnant does not collapse to black hole

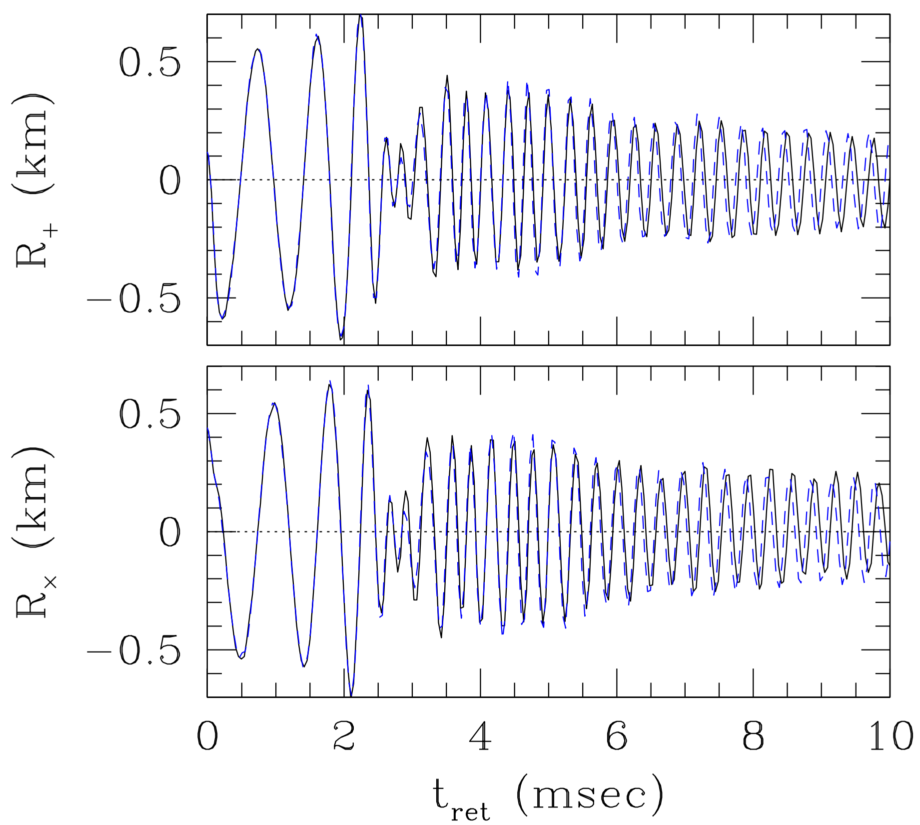
- Supported by virtue of differential rotation $\Longrightarrow$ "hypermassive" neutron stars [Baumgarte *et.al.*, 2000; Morrison *et.al.*, 2004]



[Shibata *et.al.*, 2005]

Gravitational Wave Signal

Compute gravitational wave signal from quadrupolar Moncrief variables



[Shibata *et.al.*, 2005]

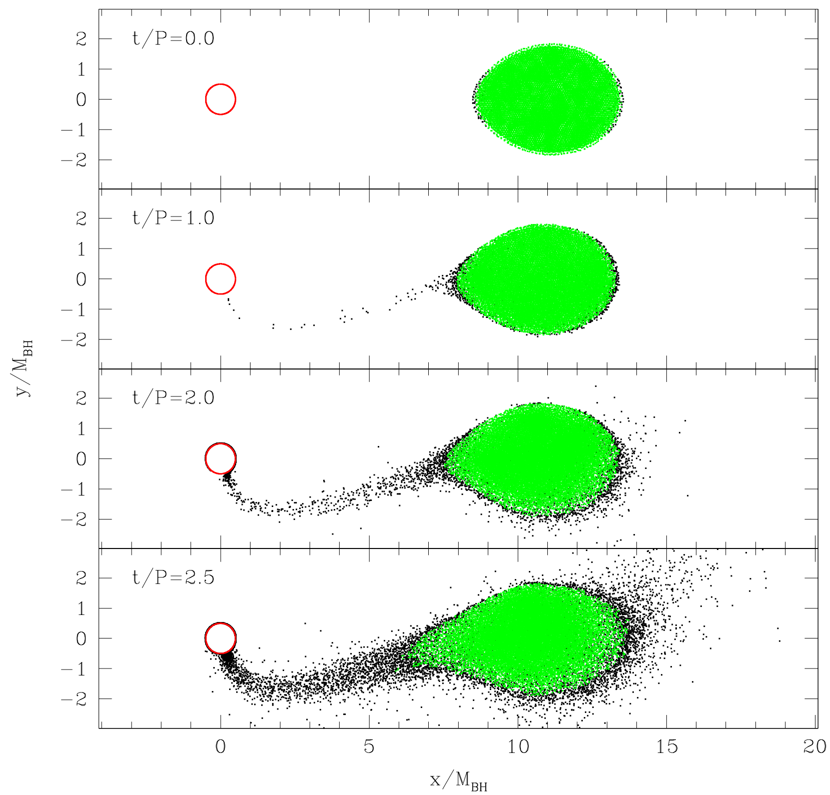
Tidal disruption of neutron star

- Initial data: quasiequilibrium model of black hole-neutron star binary [Baumgarte *et.al.*, 2004]

- assume $M_{\rm BH} \gg M_{\rm NS}$
$\Longrightarrow$ can remove black hole from computational grid

- $n = 1$ polytrope
$\Longrightarrow$ stable accretion onto black hole

- evolve with SPH code (compare with semi-analytic predictions)



[Faber *et.al.*, in prep]

Beyond perfect fluids: viscosity

Include viscosity term

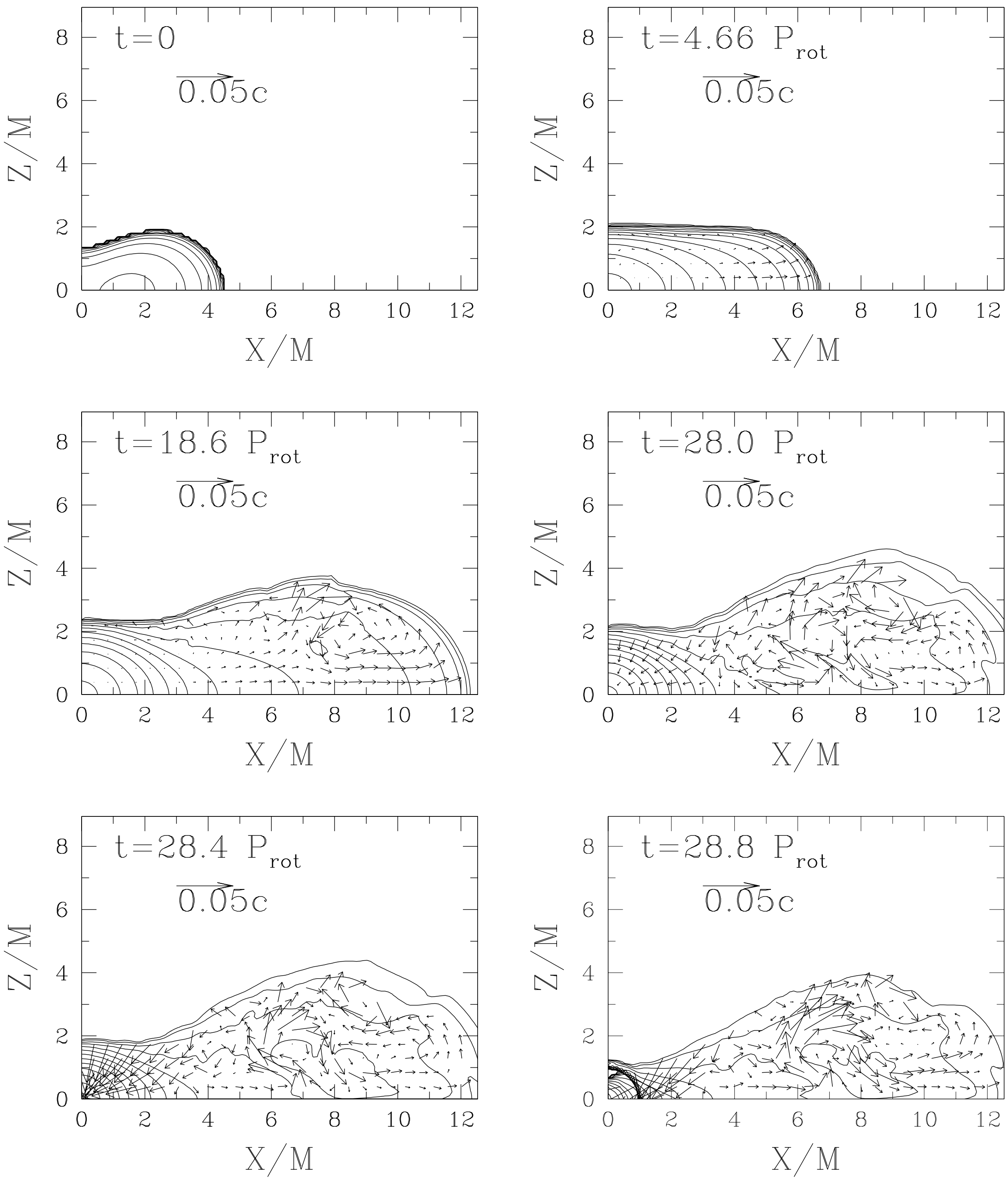
$$T_{ab} = \rho_0 h u_a u_b + P g_{ab} - 2\eta \sigma_{ab},$$

where η is coefficient of viscosity and σ_{ab} is shear.

Application: study evolution of hypermassive stars:

- viscosity reduces degree of differential rotation
- decreases rotation at core
  $\Longrightarrow$ collapse
- increases rotation at equator
  $\Longrightarrow$ expansion

[Duez *et.al.*, 2004]



t=0
0.05c
t=4.66 P_{rot}
0.05c
t=18.6 P_{rot}
0.05c
t=28.0 P_{rot}
0.05c
t=28.4 P_{rot}
0.05c
t=28.8 P_{rot}
0.05c
Z/M
X/M

Beyond perfect fluids: Magnetohydrodynamics (MHD)

- Additional stress-energy tensor includes

$$T^{ab}_{\rm em} = \frac{1}{4\pi}\left(F^{ac}F^{b}{}_{c} - \frac{1}{4}g^{ab}F_{cd}F^{cd}\right),$$

express in terms of E^a and B^a

- Ideal MHD condition $E^a_{(u)} = 0$ yields

$$\partial_t \mathcal{B}^i + \partial_j(v^j\mathcal{B}^i - v^i\mathcal{B}^j) = 0$$

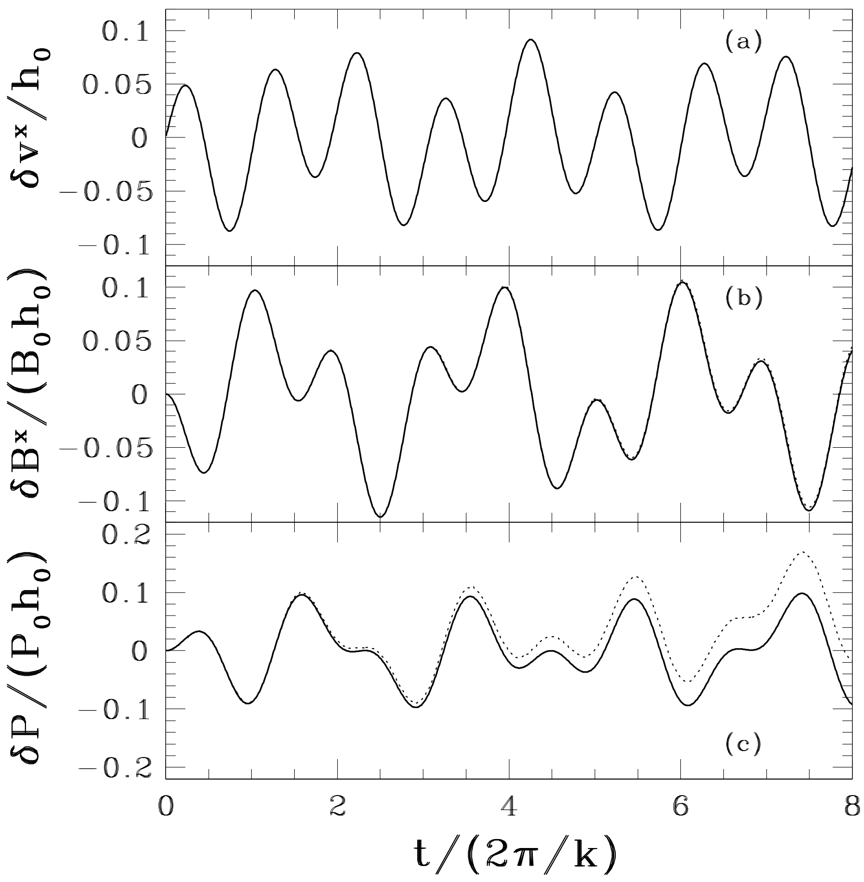
where $\mathcal{B}^i = \sqrt{\gamma}B^i$

- Evolve together with hydrodynamics and gravitational fields

[Duez *et.al.*, 2005a]

Gravitational Wave-Induced MHD Waves

- Example of relativistic MHD solution in dynamical background
- Solution analytic to linear order [Duez *et.al.*, 2005b]



[Duez *et.al.*, 2005a]

Summary

- Neutron star simulations in pretty good shape
- Can perform simulations of astrophysical interest
- Lots of room for improvements, but no show-stopper