Critical Collapse in Newtonian Self-Gravitating Systems

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Outline

- Black holes and singularities
- Overview of critical collapse
- Critical Collapse in Newtonian Phenomena
- Shed light on problems using Newtonian gravity
- Proposed research

Black Holes

Schwarzschild solution (1917)

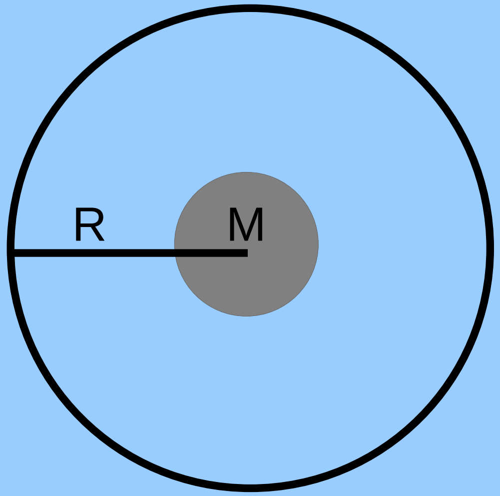
Can use Newtonian reasoning to guess the Schwarzschild radius

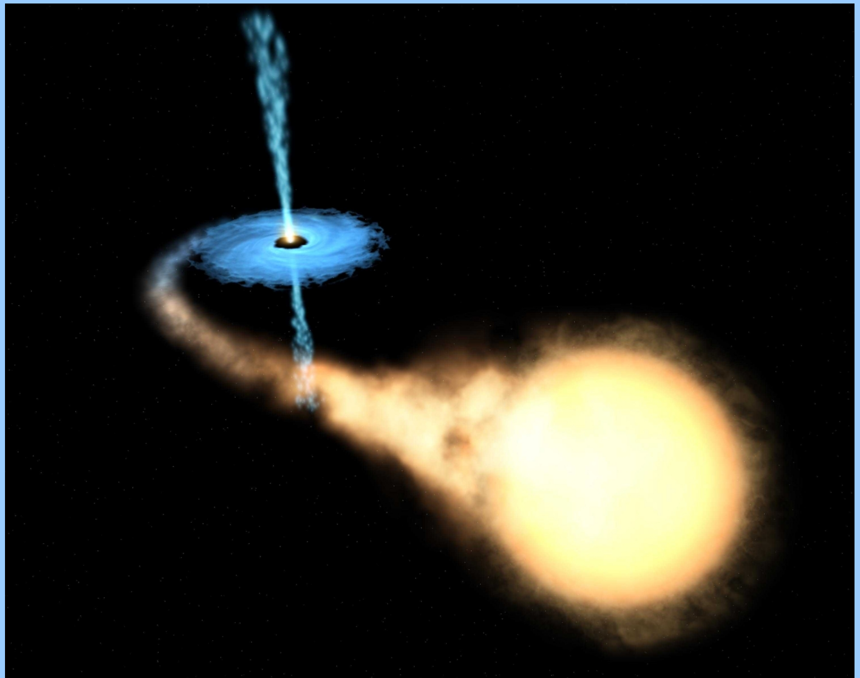
Escape velocity

$$v_e = \sqrt{\frac{2GM}{R}}$$

$$R = 2M$$

$$c = G = 1$$

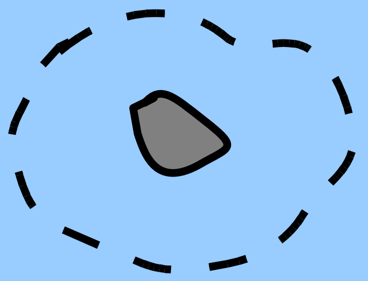




Singularities

Penrose-Hawking Singularity Theorems (1960's)

Guarantee singularity formation of sufficiently dense mass and energy configurations.



Sufficiently weak gravitating systems **never** become singular

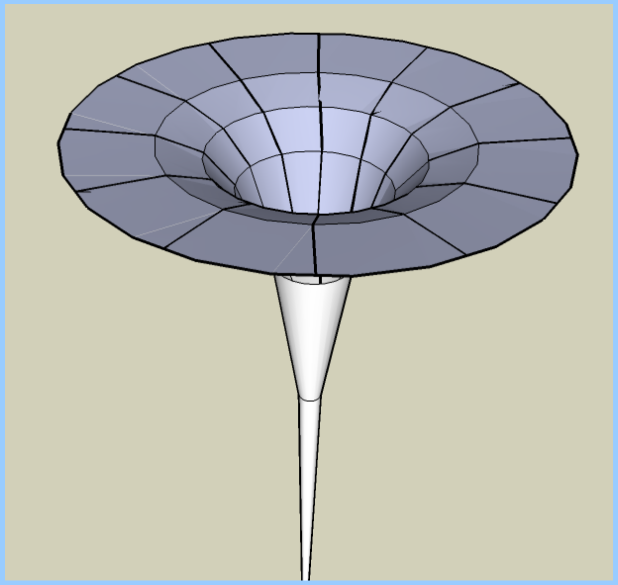
•No sufficient condition for black hole formation found beyond singularity theorems.

•Dynamics of black hole formation is not well understood

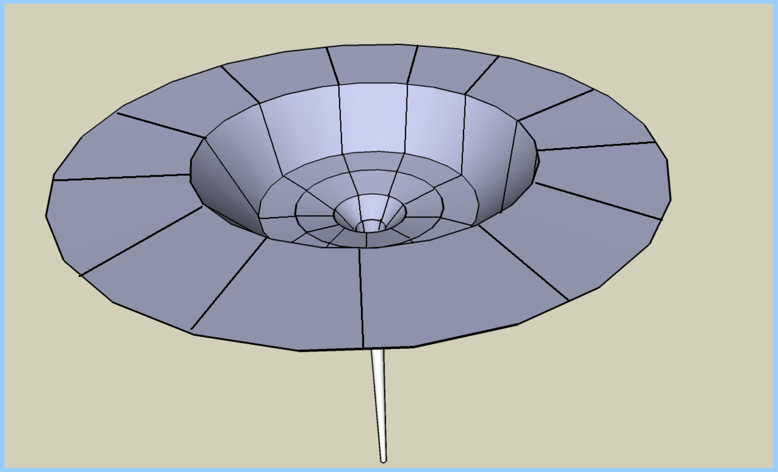
Critical Collapse

Choptuik (1990s)

Used massless scalar field coupled to gravity in numerical studies



$$p < p^*$$



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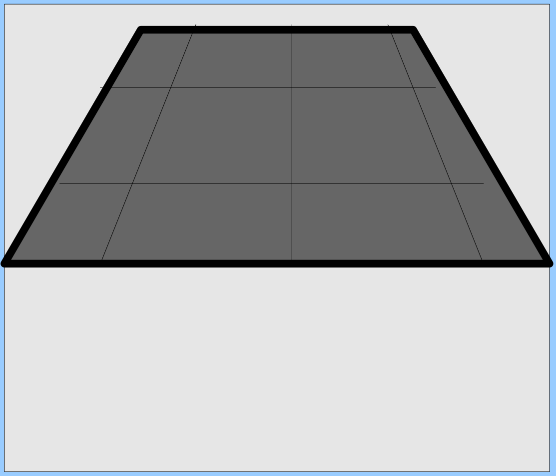
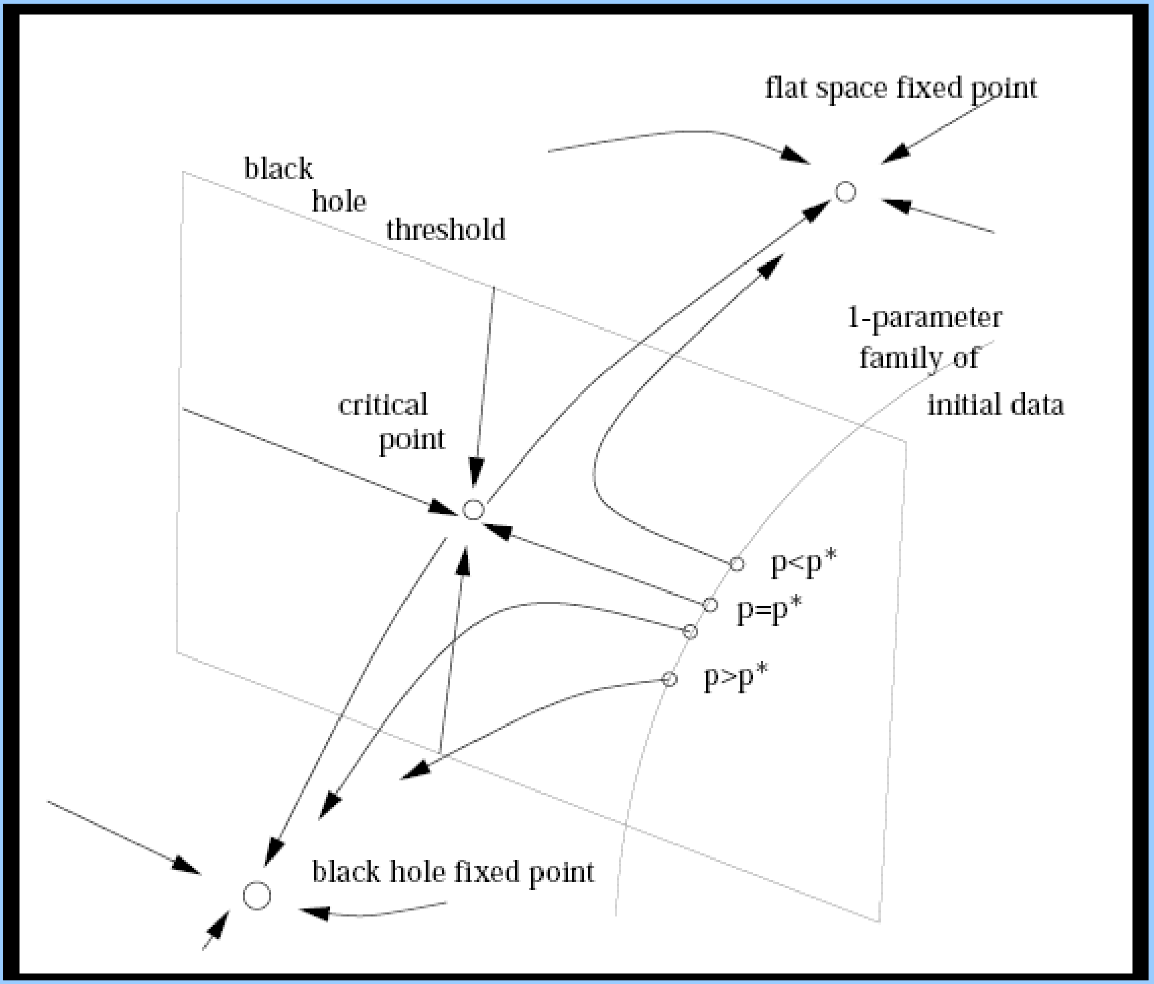


$$p > p^*$$

Phase Diagram



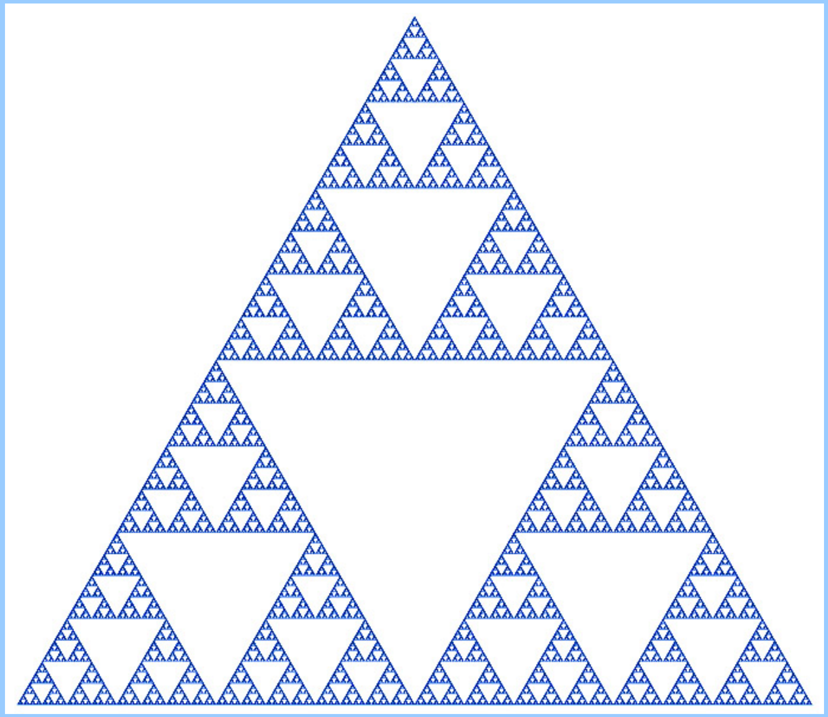
Gundlach, 2002

Analogy to Phase Transitions in Statistical Mechanics

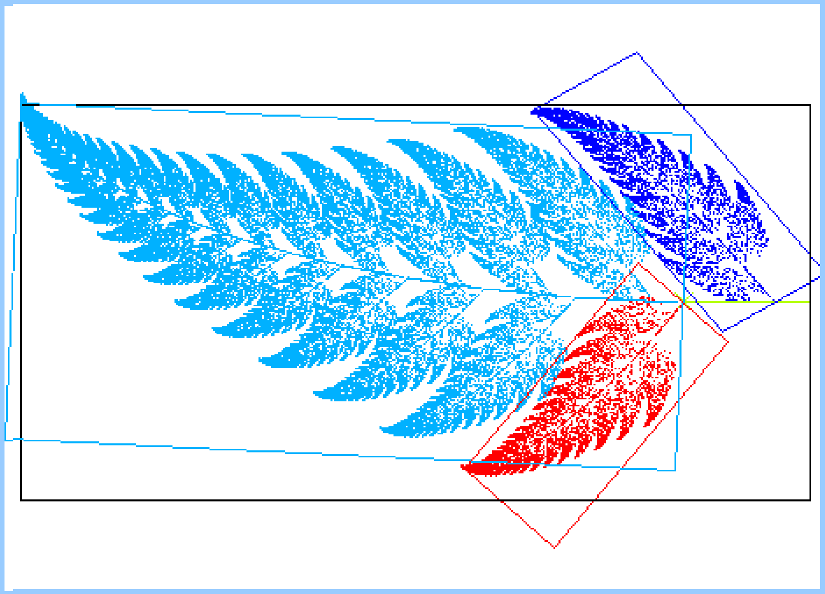
Type I	Type II
•Occur when a mass scale is set •Mass of final states independent of $p - p^*$ •Critical solution is independent of time, or periodic in time.	•Mass of final state obeys a power-law scaling $$M \propto (p - p^*)^\gamma$$ •Critical solution is self-similar and scale-invariant (continuously or discretely)

Self-Similarity

Scale-invariance



Sierpinski Triangle



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Critical Solutions self-similar in logarithmic time

Newtonian Critical Collapse

•Matter models (Newtonian isothermal gas, massive scalar fields)

•Also exhibit self-similar solutions
(Hunter and Larson-Penston)

•Important in understanding star formation in interstellar clouds

•Advantageous for investigating challenging problems in critical collapse

Self-Similarity Hypothesis

- Self-similarity also found in non-critical solution in collapse of gas models.

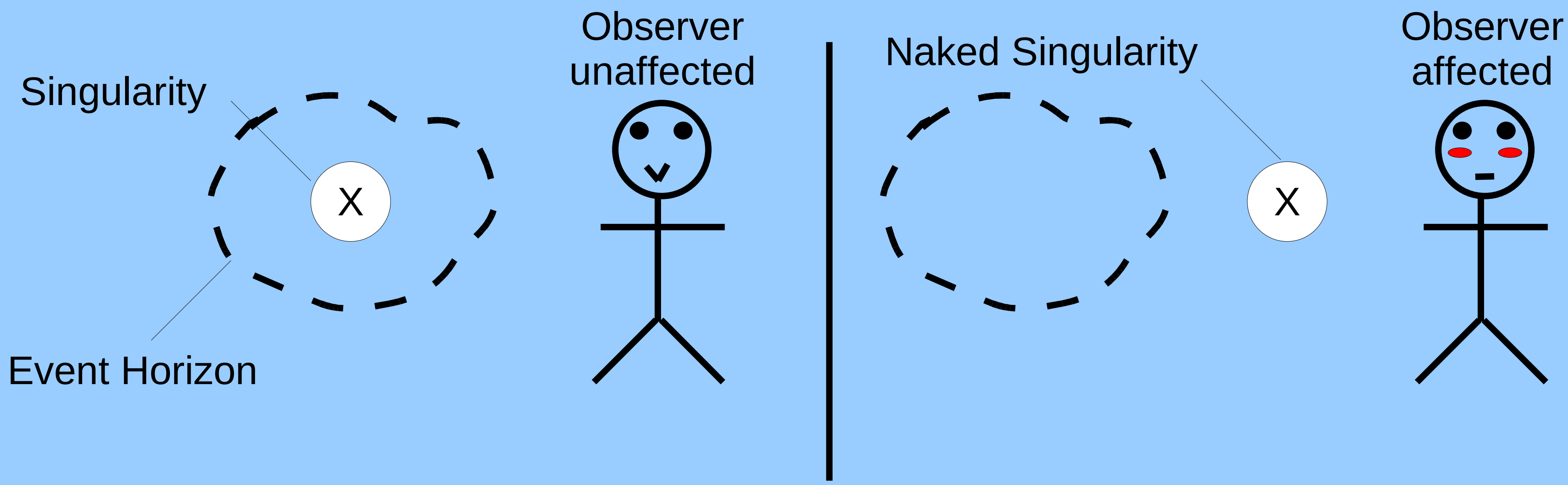
- Hypothesized that **all** initial configurations will evolve towards SS.

- Need more studies beyond spherical symmetry

Cosmic Censorship

Conjecture: Singularities are always enclosed by an event horizon.

Is key to determining the precise range of validity of GR



Is formation of naked singularities stable against non-spherical perturbation?

Can dynamics tell us more about formation of naked singularities?

Matter Model

Complex massive scalar field in spherical symmetry.

Relativistic equations of motion reduce to a coupled system of a Schrodinger equation and a Poisson equation.

$$i\frac{\partial \Psi}{\partial t} = -\frac{1}{2}\nabla^2\Psi + V\Psi$$

$$\nabla^2 V = \Psi\Psi^*$$

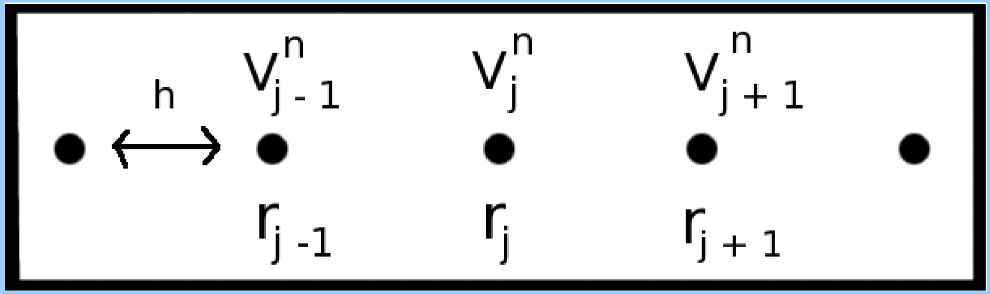
$$c = G = \hbar = 1$$

Numerical Techniques

Finite-differencing

$$\nabla^2 V = \frac{2}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial r^2}$$

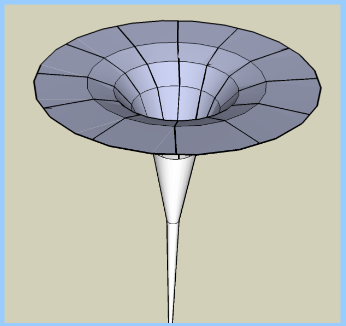
$$\mathcal{L}\mathbf{V} = \mathbf{\Psi}\mathbf{\Psi}^*$$



Crank-Nicholson is a popular scheme to solve Schrodinger equations.

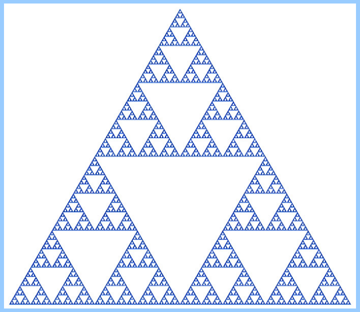
Research Goals

•Look for critical phenomena in complex massive scalar field

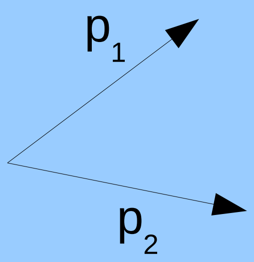


•Look for self-similarity

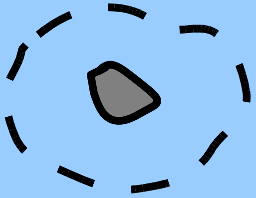
➢Calculate exact form of solution



•Investigations with more parameters



•Beyond spherical symmetry



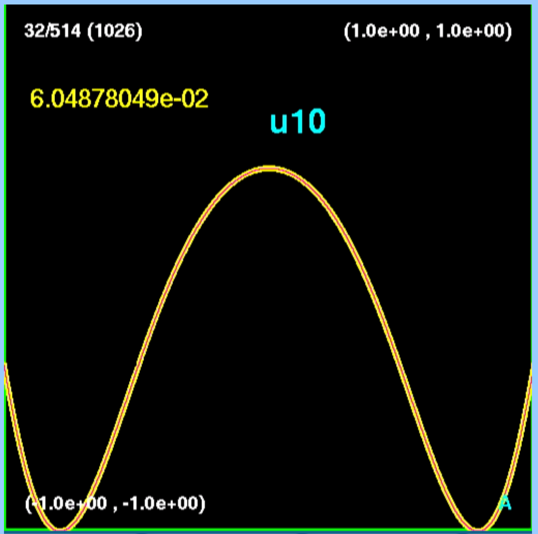
Resources

•For initial exploratory calculations, only require one processor.

•If singularities and SS found, require adaptive mesh refinement techniques = more expensive

- Time on computer clusters at UBC will be needed.

•Software for visualization of time-dependent PDEs developed by Choptuik (RNPL and XVS)



XVS output

Summary

Gravitational collapse will be modeled using a complex massive scalar field

Hope to find:

- Critical Phenomena
- Self-similarity

Hope to gain insights into:

- Dynamics of black hole formation
- Validity of self-similarity hypothesis
- Validity of cosmic censorship conjecture