A Numerical Study of Boson Star Binaries

2nd PhD Committee Meeting

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Outline

- Motivation
- Matter Model: Scalar Field
- Boson Stars in Spherical Symmetry
- Conformal Flat Approximation
- Current Project: Coalescence of Boson Stars

Motivation

- Why study compact binaries?
  - One of most promising sources of gravitational waves
  - It is a good laboratory to study the phenomenology of strong gravitational fields
- Why boson stars?
  - Plunge and merge phase of the inspiral of compact objects is characterized by a strong dynamical gravitational field. In this regime gross features of fluid and boson stars' dynamics may be similar
  - Since the details of the dynamics of the stars (e.g. shocks) tend not to be important gravitationally, boson star binaries may provide some insight into NS binaries
- Development of a computational infrastructure for 3D codes
  - 3D numerical relativistic calculations are computationally very expensive. Need for more efficient computational techniques: AMR, parallelization.
    - This infrastructure is being constructed by Frans Pretorius

Matter Model: Scalar Field

- A massive complex field is chosen as matter source because it is a simple type of matter that allows a star-like solution and because there will be no problems with shocks, low density regions, ultrarelativistic flows, etc in the evolution of this kind of matter as opposed to fluids

- The matter content is described by the scalar field:

$$\Phi = \phi_1 + i\phi_2 \tag{1}$$

  where ϕ_1 and ϕ_2 are real-valued

- The Lagrangian density associated with this field is given by:

$$L_\Phi = -\frac{1}{8\pi}(g^{ab}\nabla_a\Phi\nabla_b\Phi^* + m^2\Phi\Phi^*) \tag{2}$$

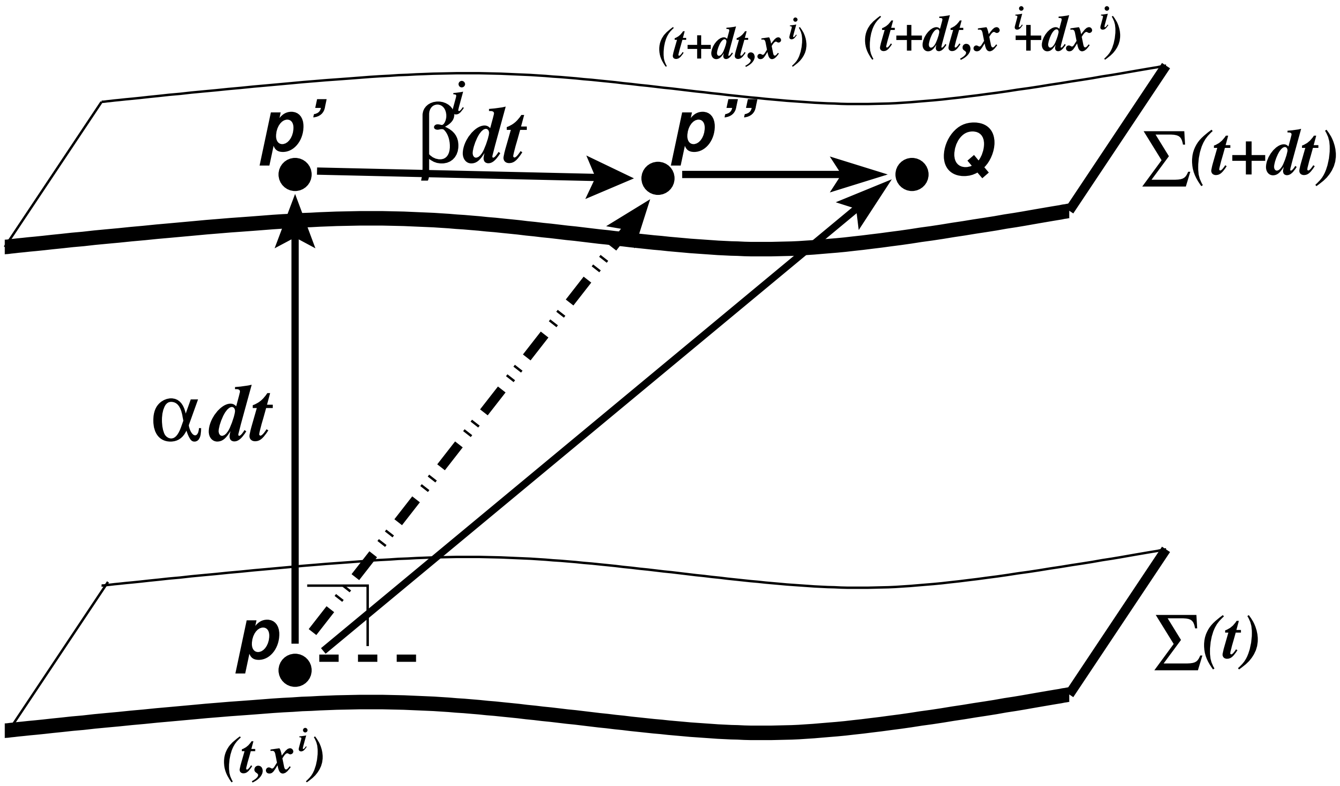
- Extremizing this action with respect to each component of the scalar field, we get the Klein-Gordon equation

$$\Box\phi_A - m^2\phi_A = 0 \qquad A = 1, 2 \tag{3}$$

Boson Stars in Spherical Symmetry

- 3+1 line element

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}\left(dx^i + \beta^i dt\right)\left(dx^j + \beta^j dt\right)$$



A schematic representation of the ADM (or 3+1) decomposition

Boson Stars in Spherical Symmetry

- From the point of view of ADM formalism the Hamiltonian formulation of the dynamics of scalar field is more useful

- The conjugate momentum field is defined as

$$\sigma_A \equiv \frac{\delta(\sqrt{-g}L_{\phi_A})}{\delta\dot{\phi}_A} \tag{4}$$

- In terms of these fields, the dynamical equations are given by

$$\partial_t \phi_A = \frac{\alpha^2}{\sqrt{-g}}\sigma_A + \beta^i \partial_i \phi_A \tag{5}$$

$$\partial_t \sigma_A = \partial_i(\beta^i \sigma_A) + \partial_i(\sqrt{-g}\gamma^{ij}\partial_j \phi_A) - \sqrt{-g}m^2 \phi_A \tag{6}$$

Boson Stars in Spherical Symmetry

- The stress-energy tensor is given by

$$T_{ab} = -2\frac{\delta L_\Phi}{\delta g^{ab}} + g_{ab}L_\Phi \tag{7}$$

- We have the following ADM components of the stress tensor

$$\begin{aligned}
\rho &= n^\mu n^\nu T_{\mu\nu} = \frac{1}{8\pi}\sum_{A=1}^{2}\Big(\frac{\alpha^2}{(-g)}\sigma_A^2 + \gamma^{ij}\partial_i\phi_A\partial_j\phi_A + m^2\phi_A^2\Big) \\
j^i &= -n^\mu T_\mu{}^i = \frac{1}{8\pi}\sum_{A=1}^{2}\Big(-2\frac{\alpha\sigma_A}{\sqrt{-g}}\gamma^{ij}\partial_j\phi_A\Big) \\
S_{ij} &= T_{ij} \\
&= \frac{1}{8\pi}\sum_{A=1}^{2}\Big(2\partial_i\phi_A\partial_j\phi_A + \gamma_{ij}\Big[\frac{\alpha^2\sigma_A^2}{(-g)} - \gamma^{mn}\partial_m\phi_A\partial_n\phi_A - m^2\phi_A^2\Big]\Big)
\end{aligned} \tag{8}$$

Boson Stars in Spherical Symmetry

- Constraint Equations: From $G_{0i} = 8\pi T_{0i}$, which do not contain 2nd time derivatives of the γ_{ij}

- Hamiltonian Constraint

$$R + K^2 - K_{ij}K^{ij} = 16\pi\rho \tag{9}$$

where R is the 3-dim. Ricci scalar, and $K \equiv K^i{}_i$ is the mean extrinsic curvature.

- Momentum Constraint

$$D_i K^{ij} - D^j K = 8\pi j^i \tag{10}$$

- Evolution Equations: From definition of extrinsic curvature, $G_{ij} = 8\pi T_{ij}$, and Ricci's equation.

$$\mathcal{L}_t \gamma_{ij} = \mathcal{L}_\beta \gamma_{ij} - 2\alpha K_{ij} \tag{11}$$

$$\mathcal{L}_t K_{ij} = \mathcal{L}_\beta K_{ij} - D_i D_j \alpha + \alpha \left(R_{ij} + K K_{ij} - 2K_{ik}K^k{}_j \right) - 8\pi\alpha(S_{ij} - \frac{1}{2}\gamma_{ij}(S - \rho)) \tag{12}$$

Boson Stars in Spherical Symmetry

- Spherically Symmetric Spacetime (SS):

$$ds^2 = \left(-\alpha^2 + a^2\beta^2\right) dt^2 + 2a^2\beta\, dtdr + a^2 dr^2 + r^2 b^2 d\Omega^2 \,, \tag{13}$$

- Hamiltonian constraint:

$$-\frac{2}{arb}\left\{\left[\frac{(rb)'}{a}\right]' + \frac{1}{rb}\left[\left(\frac{rb}{a}(rb)'\right)' - a\right]\right\} + 4K^r{}_r K^\theta{}_\theta + 2{K^\theta{}_\theta}^2 = 8\pi\left[\frac{|\Phi|^2 + |\Pi|^2}{a^2} + m^2|\phi|^2\right] \tag{14}$$

- Momentum constraint:

$$K^\theta{}_\theta{}' + \frac{(rb)'}{rb}\left(K^\theta{}_\theta - K^r{}_r\right) = \frac{2\pi}{a}\left(\Pi^*\Phi + \Pi\Phi^*\right) . \tag{15}$$

where the auxiliary field variables were defined as:

$$\Phi \equiv \phi' \,, \tag{16}$$

$$\Pi \equiv \frac{a}{\alpha}\left(\dot{\phi} - \beta\phi'\right) , \tag{17}$$

Boson Stars in Spherical Symmetry

- Evolution equations

$$\dot{a} = -\alpha a K^r{}_r + (a\beta)' \tag{18}$$

$$\dot{b} = -\alpha b K^\theta{}_\theta + \frac{\beta}{r}(rb)' \,. \tag{19}$$

$$\dot{K^r{}_r} = \beta K^r{}_r{}' - \frac{1}{a}\left(\frac{\alpha'}{a}\right)' + \alpha\left\{-\frac{2}{arb}\left[\frac{(rb)'}{a}\right]' + K K^r{}_r - 4\pi\left[\frac{2|\Phi|^2}{a^2} + m^2|\phi|^2\right]\right\} \tag{20}$$

$$\dot{K^\theta{}_\theta} = \beta K^\theta{}_\theta{}' + \frac{\alpha}{(rb)^2} - \frac{1}{a(rb)^2}\left[\frac{\alpha rb}{a}(rb)'\right]' + \alpha\left(K K^\theta{}_\theta - 4\pi m^2|\phi|^2\right) \tag{21}$$

- Field evolution equations

$$\dot{\phi} = \frac{\alpha}{a}\Pi + \beta\Phi \tag{22}$$

$$\dot{\Phi} = \left(\beta\Phi + \frac{\alpha}{a}\Pi\right)' \tag{23}$$

$$\dot{\Pi} = \frac{1}{(rb)^2}\left[(rb)^2\left(\beta\Pi + \frac{\alpha}{a}\Phi\right)\right]' - \alpha a m^2\phi + 2\left[\alpha K^\theta{}_\theta - \beta\frac{(rb)'}{rb}\right]\Pi \tag{24}$$

Boson Stars in Spherical Symmetry

- Maximal-isotropic coordinates
  - Maximal slicing condition

$$K \equiv K_i^i = 0 \qquad\qquad \dot{K}(t,r) = 0 \tag{25}$$

  - Isotropic condition

$$a = b \equiv \psi(t,r)^2 \tag{26}$$

  - They fix the lapse and shift (equivalent of fixing the coordinate system)

$$\alpha'' + \frac{2}{r\psi^2}\frac{d}{dr^2}\left(r^2\psi^2\right)\alpha' + \left[4\pi\psi^4 m^2|\phi|^2 - 8\pi|\Pi|^2 - \frac{3}{2}\left(\psi^2 {K^r}_r\right)^2\right]\alpha = 0 \tag{27}$$

$$r\left(\frac{\beta}{r}\right)' = \frac{3}{2}\alpha {K^r}_r \tag{28}$$

  - Constraint equations

$$\frac{3}{\psi^5}\frac{d}{dr^3}\left(r^2\frac{d\psi}{dr}\right) + \frac{3}{16}{K^r}_r{}^2 = -\pi\left(\frac{|\Phi|^2+|\Pi|^2}{\psi^4} + m^2|\phi|^2\right) \tag{29}$$

$${K^r}_r{}' + 3\frac{(r\psi^2)'}{r\psi^2}{K^r}_r = -\frac{4\pi}{\psi^2}\left(\Pi^*\Phi + \Pi\Phi^*\right) \tag{30}$$

Boson Stars in Spherical Symmetry

- Complex-scalar field evolution equations

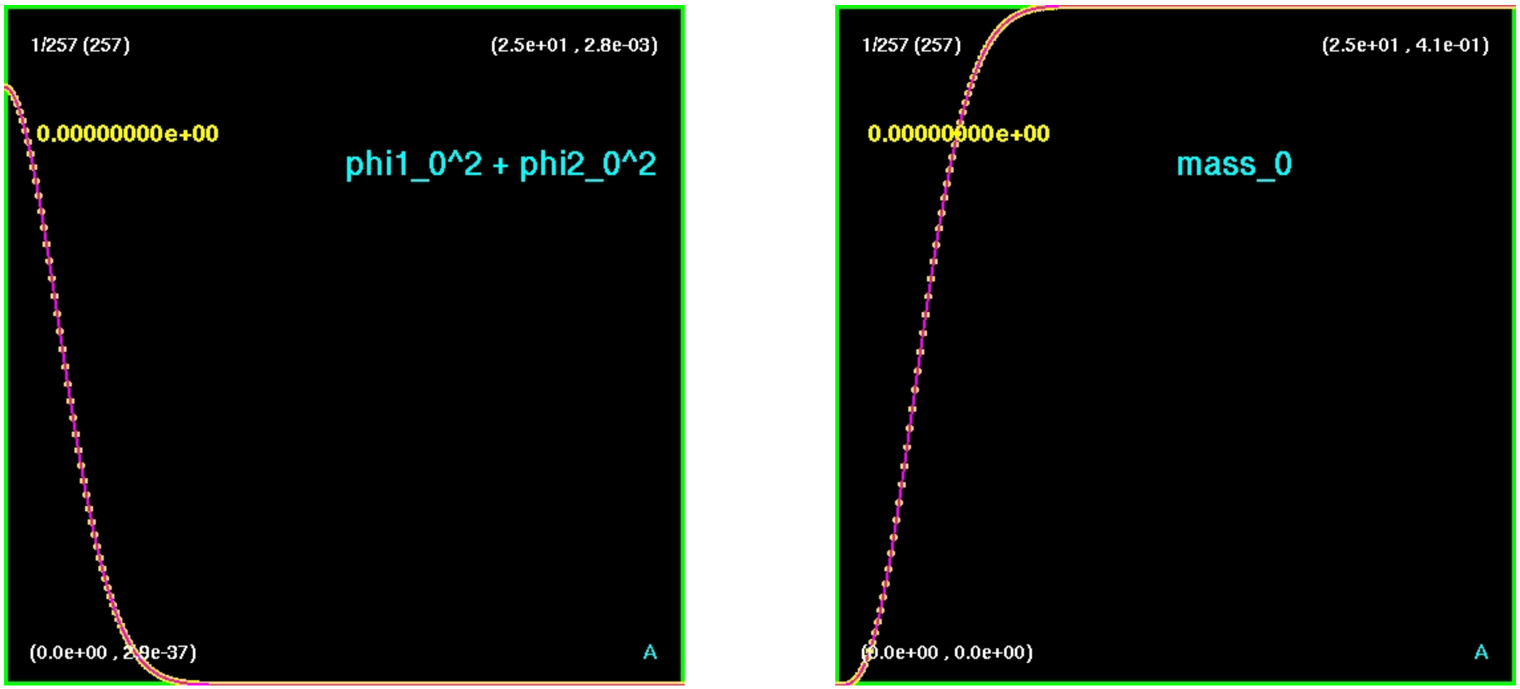
$$\dot{\phi} = \frac{\alpha}{\psi^2}\Pi + \beta\Phi \tag{31}$$

$$\dot{\Phi} = \left(\beta\Phi + \frac{\alpha}{\psi^2}\Pi\right)' \tag{32}$$

$$\dot{\Pi} = \frac{3}{\psi^4}\frac{d}{dr^3}\left[r^2\psi^4\left(\beta\Pi + \frac{\alpha}{\psi^2}\Phi\right)\right] - \alpha\psi^2 m^2\phi - \left[\alpha K^r{}_r + 2\beta\frac{(r\psi^2)'}{r\psi^2}\right]\Pi \tag{33}$$

Boson Stars in Spherical Symmetry

- These equations were coded using RNPL and tested for a gaussian pulse as initial data.



Boson Stars in Spherical Symmetry

- Initial Value Problem

- We are interested in generating *static* solutions of the Einstein- Klein-Gordon system

- There is no regular, time-independent configuration for complex scalar fields but one can construct harmonic time-dependence that produce time-independ ent metric

- We adopt the following ansatz for boson stars in spherical symmetry in order to produce a static spacetime:

$$\phi(t,r) = \phi_0(r)\, e^{-i\omega t}\,, \qquad\qquad \beta = 0 \tag{34}$$

  where the last condition comes from the demand of a static timelike Killing vector field.

- Polar-Areal coordinates

$$K = K^r{}_r \qquad\qquad b = 1 \tag{35}$$

- Generalization of the usual Schwarzschild coordinates to *time-dependent*, spherically symmetric spacetimes. Easier to generate the initial data solution

Boson Stars in Spherical Symmetry

- The line element

$$ds^2 = -\alpha^2 dt^2 + a^2 dr^2 + r^2 d\Omega^2 \,. \tag{36}$$

- The equations of motions are cast in a system of ODEs. It becomes an eigenvalue problem with eigenvalue $\omega = \omega(\phi_0(0))$

$$a' = \frac{1}{2}\left\{\frac{a}{r}\left(1-a^2\right) + 4\pi r a\left[\phi^2 a^2\left(m^2 + \frac{\omega^2}{\alpha^2}\right) + \Phi^2\right]\right\} \tag{37}$$

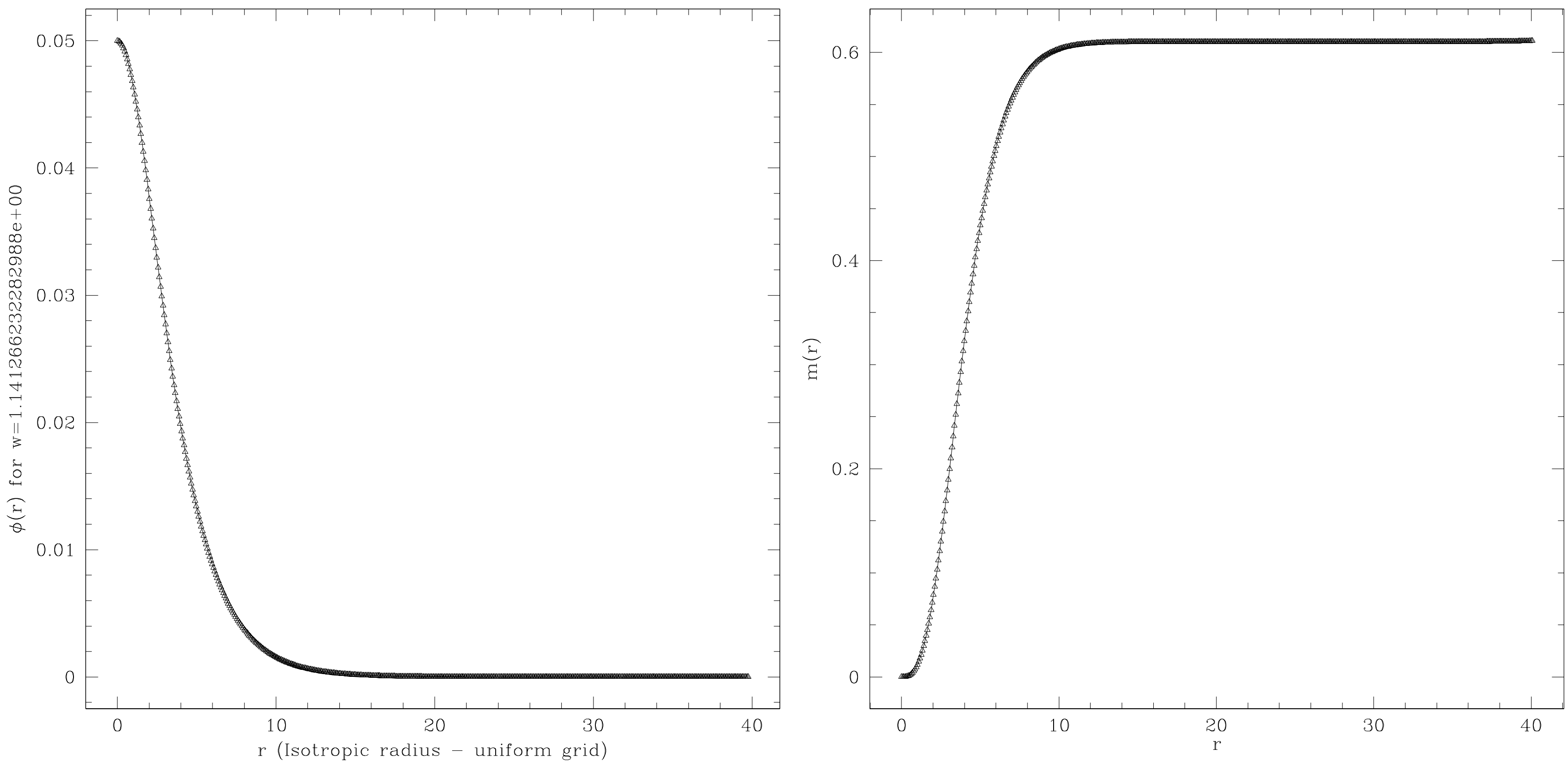
$$\alpha' = \frac{\alpha}{2}\left\{\frac{a^2-1}{r} + 4\pi r\left[a^2\phi^2\left(\frac{\omega^2}{\alpha^2} - m^2\right) + \Phi^2\right]\right\} \tag{38}$$

$$\phi' = \Phi \tag{39}$$

$$\Phi' = -\left(1 + a^2 - 4\pi r^2 a^2 m^2 \phi^2\right)\frac{\Phi}{r} - \left(\frac{\omega^2}{\alpha^2} - m^2\right)\phi a^2 \tag{40}$$

Boson Stars in Spherical Symmetry

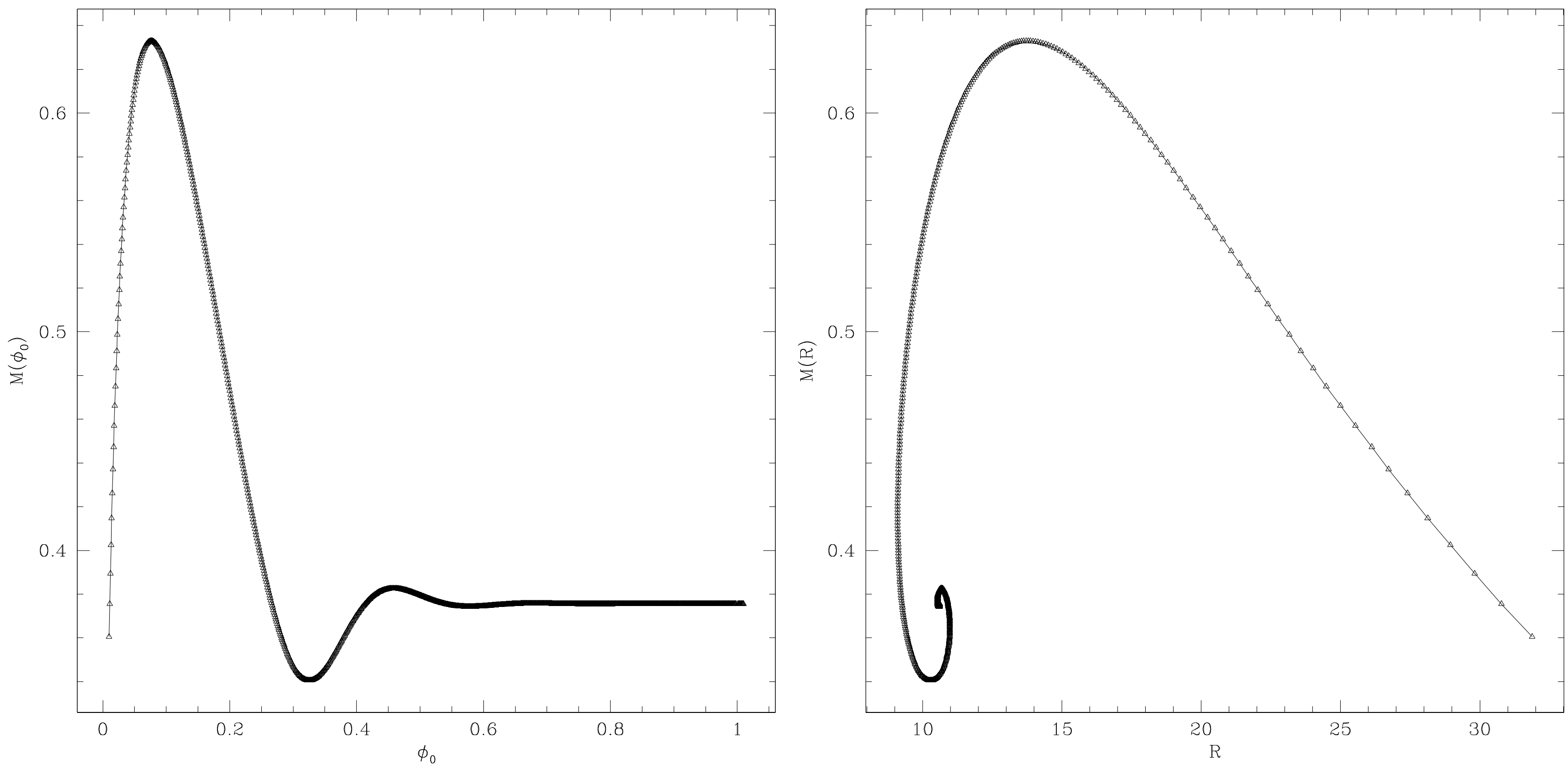
- Field configuration and its aspect mass function for $\phi_0(0) = 0.05$. Its eigenvalue was "shooted" to be $\omega = 1.1412862322$



- Note its exponentially decaying tail as opposed to the sharp edge ones for its fluids counterparts

Boson Stars in Spherical Symmetry

- The ADM mass as a function of the central density and the radius of the star as a function of ADM mass. Note their similarity to the fluid stars



Conformal Flat Approximation (CFA)

- Motivation
  - Full 3D Einstein equations are very complex and computationally expensive to solve
  - Heuristic assumption that the dynamical degrees of freedom of the gravitational fields, i.e. the gravitational radiation, play a small role in at least some phases of the strong field interaction of a merging binary
  - Gravitational radiation is small in most systems studied so far
  - CFA effectively eliminates the two dynamical degrees of freedom, simplifies the equations and allows a fully constrained evolution
  - CFA allows us to investigate the same kind of phenomena observed in the full relativistic case, such as the description of compact objects and the dynamics of their interaction; black hole formation; critical phenomena
  - CFA has been used in the past with promising results in certain cases (Wilson-Matthews studies of coalescence of neutron stars; Bruno Rousseau's master's thesis)

Conformal Flat Approximation (CFA)

- Formalism
  - The CFA prescribes a conformally flat spatial metric at all times
  - Introduce a flat metric f_{ij} as a base / background metric:

$$\gamma_{ij} = \psi^4 f_{ij} \tag{41}$$

where the conformal factor ψ is a positive scalar function describing the ratio between the scale of distance in the curved space and flat space($f_{ij} \equiv \delta_{ij}$ in cartesian coordinates)

  - In this approximation all of the geometric variables can be computed from the constraints as well as from a specific choice of coordinates
  - Maximum slicing condition is used to fix the time coordinate

$$K^i_i = 0$$
$$\partial_t K^i_i = 0 \tag{42}$$

Conformal Flat Approximation (CFA)

- Slicing Condition
  - Gives an elliptic equation for the lapse function α

$$\nabla^2 \alpha = -\frac{2}{\psi} \vec{\nabla}\psi \cdot \vec{\nabla}\alpha + \alpha\psi^4 \left(K_{ij}K^{ij} + 4\pi \left(\rho + S\right)\right) \tag{43}$$

- Hamiltonian Constraint
  - Gives an elliptic equation for the conformal factor ψ

$$\nabla^2 \psi = -\frac{\psi^5}{8} \left(K_{ij}K^{ij} + 16\pi\rho \right) \tag{44}$$

- Momentum Constraints
  - Given elliptic equations for the shift vector components β^i

$$\begin{aligned} \nabla^2 \beta^j \quad = \quad & -\frac{1}{3}\hat{\gamma}^{ij}\partial_i \left(\vec{\nabla} \cdot \vec{\beta} \right) + \alpha\psi^4 16\pi J^j - \partial_i \left[ln \left(\frac{\psi^6}{\alpha} \right) \right] \left[\hat{\gamma}^{ik}\partial_k \beta^j \right. \\ & \left. + \hat{\gamma}^{jk}\partial_k \beta^i - \frac{2}{3}\hat{\gamma}^{ij} \left(\vec{\nabla} \cdot \vec{\beta} \right) \right] \end{aligned} \tag{45}$$

Conformal Flat Approximation (CFA)

- Note that $K_{ij}K^{ij}$ can also be expressed in terms of the flat operators. It ends up being expressed as flat derivatives of the shift vector:

$$K_{ij}K^{ij} = \frac{1}{2\alpha^2}\left(\hat{\gamma}_{kn}\hat{\gamma}^{ml}\hat{D}_m\beta^k\hat{D}_l\beta^n + \hat{D}_m\beta^l\hat{D}_l\beta^m - \frac{2}{3}\hat{D}_l\beta^l\hat{D}_k\beta^k\right) \tag{46}$$

- Cartesian Coordinates

$$\frac{\partial^2\alpha}{\partial x^2}+\frac{\partial^2\alpha}{\partial y^2}+\frac{\partial^2\alpha}{\partial z^2} = -\frac{2}{\psi}\left[\frac{\partial\psi}{\partial x}\frac{\partial\alpha}{\partial x}+\frac{\partial\psi}{\partial y}\frac{\partial\alpha}{\partial y}+\frac{\partial\psi}{\partial z}\frac{\partial\alpha}{\partial z}\right]+\alpha\psi^4\left(K_{ij}K^{ij}+4\pi\left(\rho+S\right)\right) \tag{47}$$

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2} = -\frac{\psi^5}{8}\left(K_{ij}K^{ij}+16\pi\rho\right) \tag{48}$$

Conformal Flat Approximation (CFA)

- x component of the shift vector in cartesian coordinates

$$\begin{aligned}\frac{\partial^2\beta^x}{\partial x^2}+\frac{\partial^2\beta^x}{\partial y^2}+\frac{\partial^2\beta^x}{\partial z^2} \quad = \quad & -\frac{1}{3}\frac{\partial}{\partial x}\left(\frac{\partial\beta^x}{\partial x}+\frac{\partial\beta^y}{\partial y}+\frac{\partial\beta^z}{\partial z}\right)+\alpha\psi^4\,16\pi J^x \\ & -\frac{\partial}{\partial x}\left[ln\left(\frac{\psi^6}{\alpha}\right)\right]\left[\frac{4}{3}\frac{\partial\beta^x}{\partial x}-\frac{2}{3}\left(\frac{\partial\beta^y}{\partial y}+\frac{\partial\beta^z}{\partial z}\right)\right] \\ & -\frac{\partial}{\partial y}\left[ln\left(\frac{\psi^6}{\alpha}\right)\right]\left[\frac{\partial\beta^x}{\partial y}+\frac{\partial\beta^y}{\partial x}\right] \\ & -\frac{\partial}{\partial z}\left[ln\left(\frac{\psi^6}{\alpha}\right)\right]\left[\frac{\partial\beta^x}{\partial z}+\frac{\partial\beta^z}{\partial x}\right] \qquad (49)\end{aligned}$$

- $K_{ij}K^{ij}$ in cartesian coordinates

$$\begin{aligned}K_{ij}K^{ij} = \quad & \frac{1}{2\alpha^2}\Bigg[\left(\frac{\partial\beta^x}{\partial x}\right)^2+\left(\frac{\partial\beta^x}{\partial y}\right)^2+\left(\frac{\partial\beta^x}{\partial z}\right)^2+\left(\frac{\partial\beta^y}{\partial x}\right)^2+\left(\frac{\partial\beta^y}{\partial y}\right)^2+\left(\frac{\partial\beta^y}{\partial z}\right)^2 \\ & +\left(\frac{\partial\beta^z}{\partial x}\right)^2+\left(\frac{\partial\beta^z}{\partial y}\right)^2+\left(\frac{\partial\beta^z}{\partial z}\right)^2+\frac{\partial}{\partial x}\left(\beta^x\frac{\partial}{\partial x}+\beta^y\frac{\partial}{\partial y}+\beta^z\frac{\partial}{\partial z}\right)\beta^x \\ & +\frac{\partial}{\partial y}\left(\beta^x\frac{\partial}{\partial x}+\beta^y\frac{\partial}{\partial y}+\beta^z\frac{\partial}{\partial z}\right)\beta^y+\frac{\partial}{\partial z}\left(\beta^x\frac{\partial}{\partial x}+\beta^y\frac{\partial}{\partial y}+\beta^z\frac{\partial}{\partial z}\right)\beta^z \\ & -\frac{2}{3}\left(\frac{\partial\beta^x}{\partial x}+\frac{\partial\beta^x}{\partial x}+\frac{\partial\beta^x}{\partial x}\right)^2\Bigg] \qquad (50)\end{aligned}$$

Conformal Flat Approximation (CFA)

- Then the following set of functions completely characterize the geometry at each time slice

$$\alpha = \alpha(t, \vec{r}), \quad \psi = \psi(t, \vec{r}), \quad \beta^i = \beta^i(t, \vec{r}) \tag{51}$$

  where $\vec{r}$ depends on the coordinate choice for the spatial hypersurface

- The solution of the gravitational system under CFA and maximal slicing condition can be summarized as:
  - Specify initial conditions for the complex scalar field
  - Solve the elliptic equations for the geometric quantities on the initial slice
  - Update the matter field values to the next slice using their equation of motion
  - For the new configuration of matter fields, re-solve the elliptic equations for the geometric variables and again allow the matter fields to react and evolve to the next slice and so on

Previous Work

- Wilson, Matthews, Marronetti, (1996) Phys. Rev. D 54, 1317
  - Study of general relativistic hydrodynamics of a coalescing neutron-star binary system
  - They discuss the evidence that, for a realistic neutron-star equation of state, general relativistic effects may cause the stars to individually collapse into black holes prior to merging
  - Strong fields cause the last stable orbit (ISCO) to occur at a larger separation distance and lower frequency than previously estimated. This is important, since it places the coalescence closer to the maximum sensitivity range of the LIGO detectors and others
- Bruno Rousseau' masters thesis
  - Boson stars studied in axisymmetry under conformally flat approximation have been shown to behave similarly to the spherical solutions of the Einstein-Klein-Gordon equations under small perturbation

Current Project - Coalescence of Boson Stars

- Motivation
  - Wilson-Mathews compression effect results raised a controversy about the validity of the conformal flat approximation
  - In order to decide if CFA is a good approximation to model compact binaries it would be interesting to simulate it using a simpler model
  - Fluid stars and Boson stars have some similarity concerning the way they are modelled, e.g. both can be parametrized by their central density ρ_0 and have qualitatively similar plots of total mass vs ρ_0
  - Then in the strong field regime for the compact binary system the dynamics may not depend sensitively on the details of the model
  - Advantage of using scalar fields: no problems with shocks, evolution done by Klein-Gordon eqn, should not present any stability problem.

Current Project - Coalescence of Boson Stars

- Questions to be addressed
  - Would the individual collapse occur before merging for boson stars as well or it is model dependent?
  - How good is the approximation? How do we test if the results are close to solutions of Einstein equations?
  - Is the individual collapse a spurious result coming from CFA?
  - What is the final result of the merging? Can we compare to results from other techniques?
  - Where is the ISCO? Does this result match to the fluid star ones? Can be at least qualitatively compared?
  - How can we extract the gravitational waveforms from this system?

Current Project - Coalescence of Boson Stars

- Phases of the project
  - The final goal is to run detailed 3D simulations of boson stars in coalescence
  - Before starting the main project, small projects must be done
    - Concluded projects
      - IVP - generation of initial data for a boson star in spherical symmetry (1D code)
      - Evolution code for a boson star in spherical symmetry (1D code)
      - Multigrid techniques for solving the elliptic equations
      - Derivation of the 3+1 equations of motion in CFA under maximal slicing condition for the Einstein-Klein-Gordon system in 3d (no imposed spatial symmetries)
  - To be concluded in the near future
    - Building a unigrid, serial 3D code compatible/ready for parallelization and AMR implementation
  - Thereafter
    - Modify the code for use of parallel adaptive infrastructure that is being constructed
    - Start investigating collisions

Current Project - Coalescence of Boson Stars

- Collision phase
  - Add some features to the 3D code/equations such as:
    - Compactification of the spatial domain to allow the extraction of gravitational waves through the multipole expansion form.
    - Add a radiation back reaction term to the Klein-Gordon equation in order to allow the effects of the radiation into the dynamics of the system.