INTRODUCTION TO NUMERICAL RELATIVITY

LECTURE 1
OVERVIEW

2006 Spring School on Numerical Methods in Gravitation and Astrophysics

March 15, 2006, KIAS, Seoul Korea

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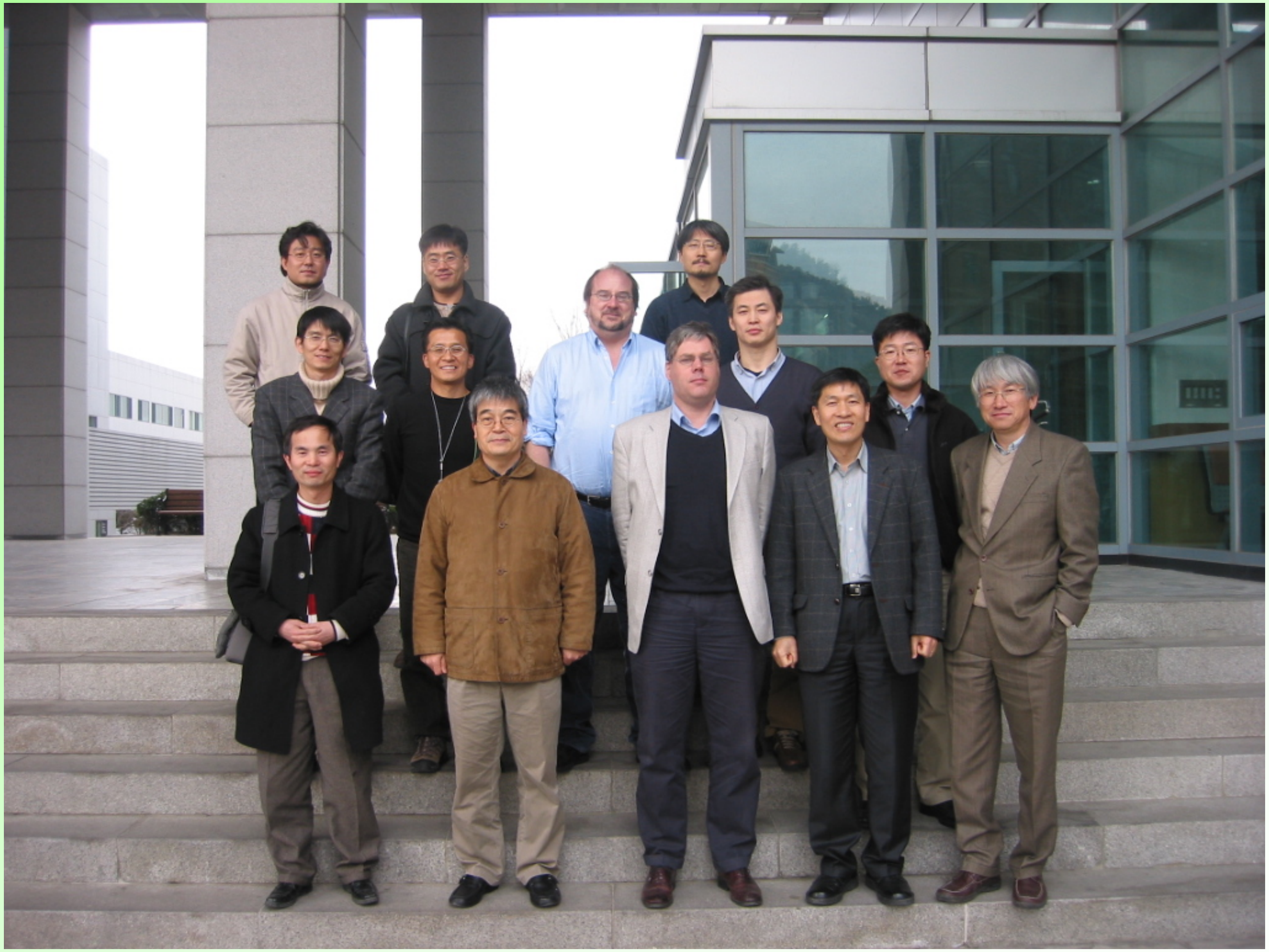
Dept of Physics & Astronomy, UBC
Vancouver, BC, Canada

(Happy 77th Birthday, Mom!)

Korea, March 2005



Isabella, Manitoba, Canada
December 2002

Vancouver 2006

Seoul
March 15 2006

LECTURE PLAN

- INTRODUCTIONS etc
- GOALS OF SPRING SCHOOL
- THE NATURE OF NUMERICAL RELATIVITY (in 2006)
- PLAN OF ATTACK
- DISCUSSION

GOALS OF SPRING SCHOOL

- Add “Korea” to the small list of names of countries that currently dominate (or historically have dominated) in the tiny field of numerical relativity (< 200 PhD theses in 30 years)

- “Easy” to do!
  - Raw materials exist in abundance here
  - View process as “technology transfer”
  - Lecturers are here to transfer knowledge & techniques to you, the students
  - If transfer isn’t going well, LET US KNOW!

The Nature of Numerical Relativity (in 2006)

- Will illustrate via 20 minute I gave repeatedly during a UBC Dept of Physics & Astronomy Open House held last year

In the beginning (@UBC Fall 1999)

In the VERY Beginning (UBC 1980-1986)

- Computer room occupied by IBM/Amdahl mainframe with up to 6 processors (Amdahl V6; our richer Albertan customers had an Amdahl V8)
- Speed was < 1 megaflop; can't remember how much (little) memory it had
- Ran typical (for the era) "Time Sharing" Operating System (MTS), which enabled literally HUNDREDS of USERS to USE it SIMULTANEOUSLY
- I marvelled when Bill Unruh let me run the multigrid solution of a 2-D elliptic equation on a 257 x 257 grid interactively; it took more than a minute real time, and cost about $50 of real grant money

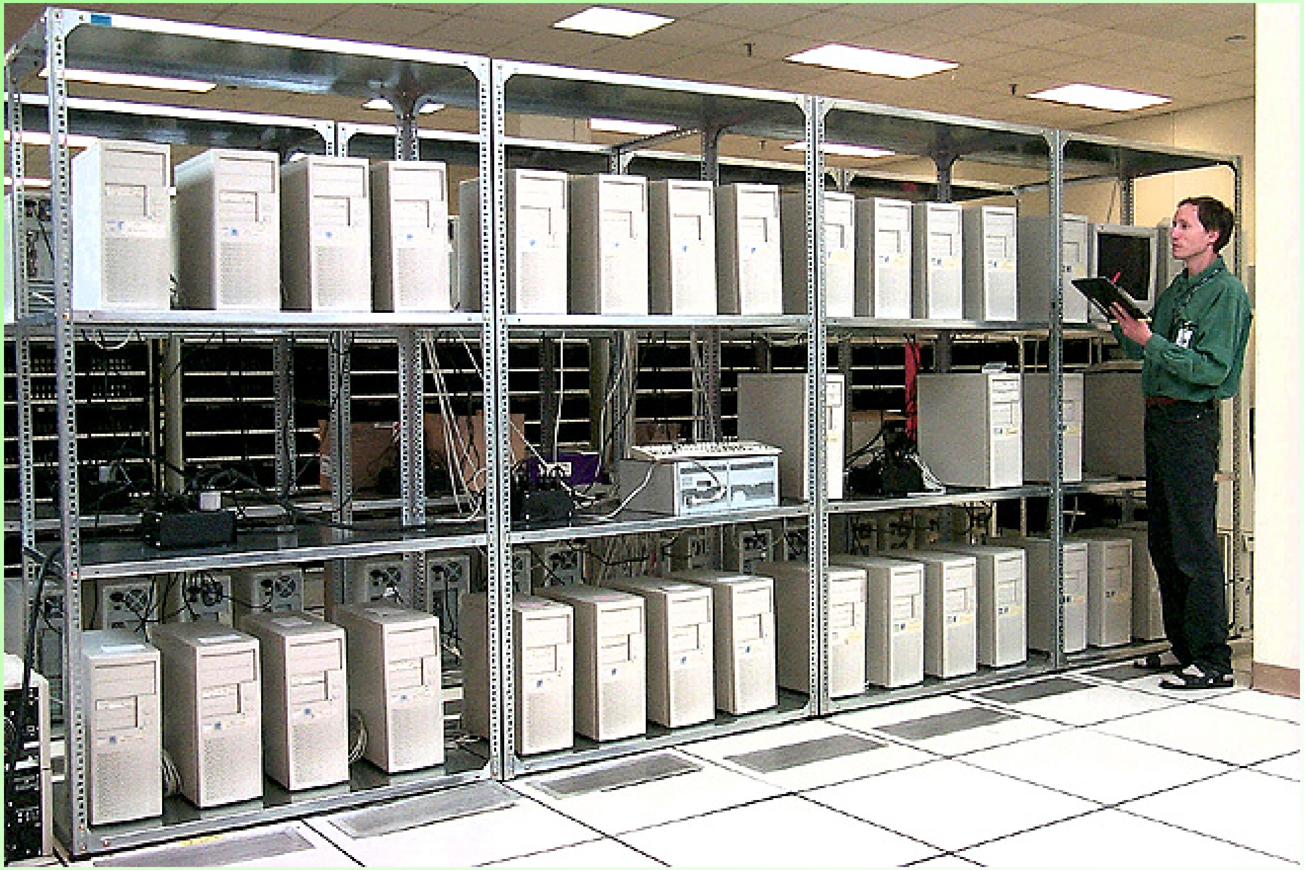
In the beginning …

Jason Ventrella, PhD UT Austin, 2002 (shown)

Inaki Olabarrietta, PhD UBC, 2004

vn.physics.ubc.ca

(UBC's first generally available supercomputing cluster)



What better home could a supercomputer want?

KUDOS!! UBC IT Services (Dave Amos, Ted Dodds, Dave Jones, Margaret Sayer, and many others!!)

Current Configuration

[CFI/ASRA/BCKDF funded HPC infrastructure]

November 1999

vn.physics.ubc.ca
128 x 0.85 GHz PIII, 100 Mbit
Up continuously since 10/98
MTBF of node: 1.9 yrs

June 2004

vnp4.physics.ubc.ca
110 x 2.4 GHz P4/Xeon, Myrinet
Up continuously since 06/03
MTBF of node: 1.9 yrs

glacier.westgrid.ca
1600 x 3.06 GHz P4, Gigiabit
Ranked #54 in Top 500 11/04 (Top in Canada)

Soon-to-be configuration

[CFI/ASRA/BCKDF funded HPC infrastructure for Joerg Rottler (UBC) and Frans Pretorius (U Alberta)]

May 2006 tentative: c3.physics.ubc.ca

- 512 Opeteron cores (128 boards)
- Myrinet 2000 Interconnect
- 512 GB RAM
- < $500K US
- Will have in excess of 50% the raw capacity of glacier.westgrid.ca, but will be fully parallel for up to n=128 cores

SO …

What do we do with these machines?

(Besides generate 100's of kW of heat!!)

Why were physicists the world over celebrating in 2005?

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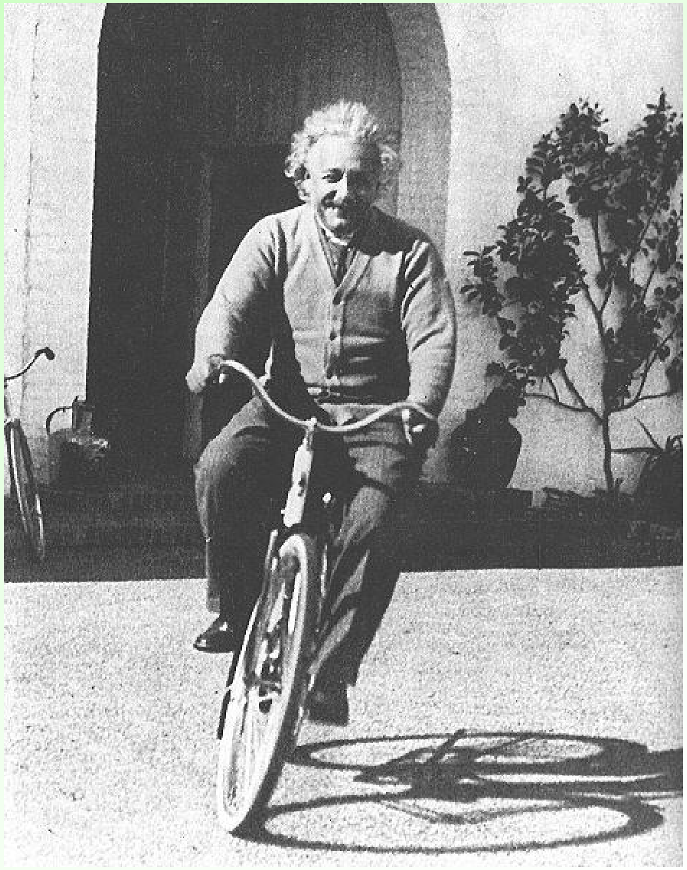
Special Relativity 1905: $E = mc^2$



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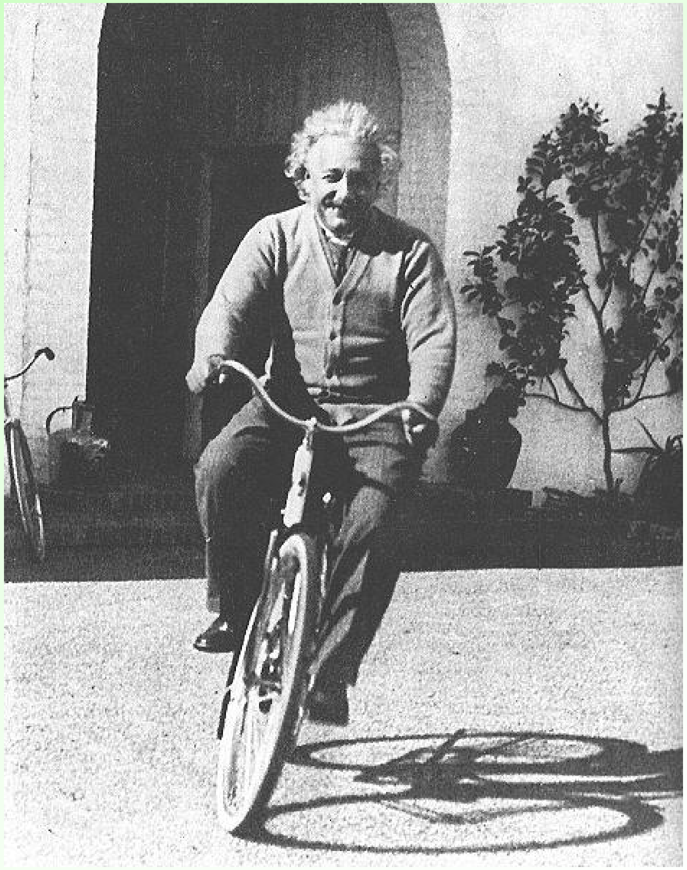

Einstein Equation(s)

GENERAL Relativity 1915-1917: $G_{\mu\nu} = 8\pi T_{\mu\nu}$

One of the things we use vn, vnp4, glacier etc. to do is to perform "simulations" by approximately solving Einstein's equations using these huge computers

Simulations of scenarios such as …

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What happens when two black holes collide?

Why should we care about colliding black holes?

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Why should we care about colliding black holes?

Gravitational wave detectors have been, and are being built!!
AND ... THEY MAY BE ABLE TO DETECT GRAVITATIONAL WAVES FROM BLACK HOLE COLLISIONS!

Images copyright LIGO consortium

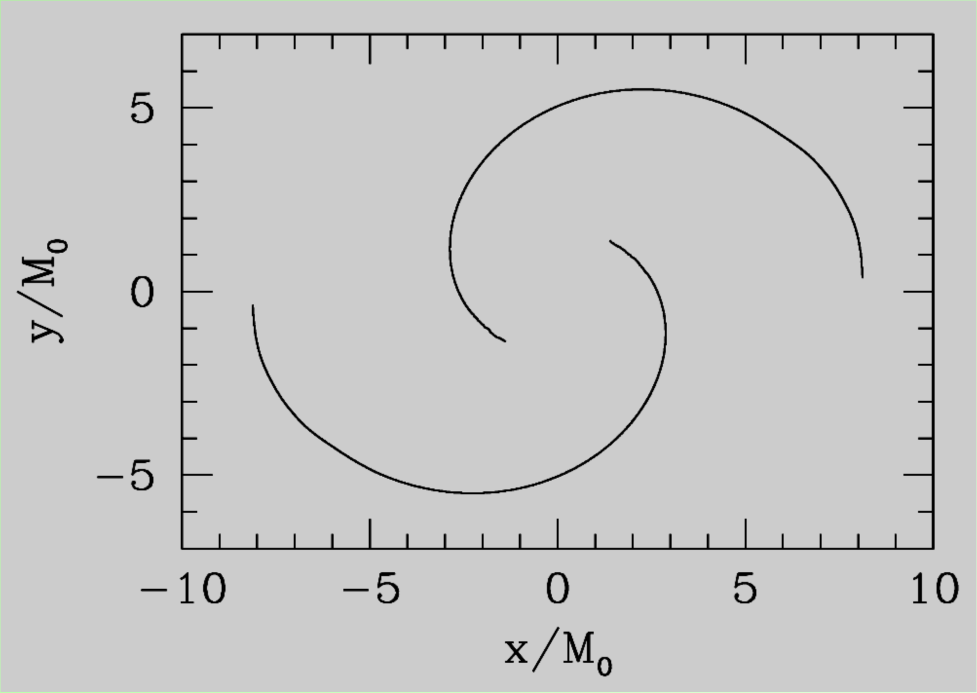
The Laser Interferometer Gravitational Wave Observatory (LIGO)

Sites in Livingston, LA and Hanford WA

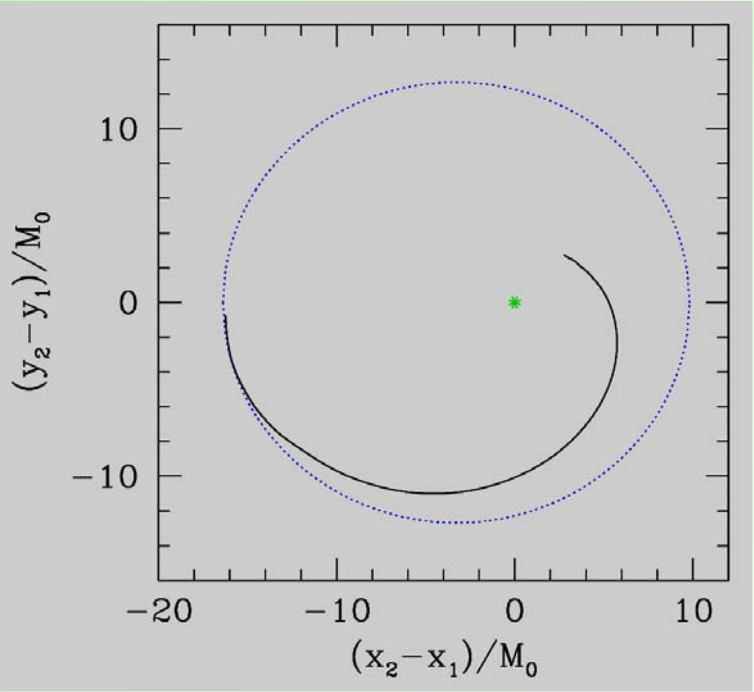
State-of-the-art calculation of black hole collision

- All calculations carried out by
  - Frans Pretorius
    - B.Eng., U. Vic., 1999
    - PhD, UBC, 2002
    - Currently R.C. Tolman Prize Postdoctoral Fellow at Caltech
    - Has just startedTier II Canada Research Chair (Asst Prof) in the Dept. of Physics, U. Alberta

Case 1: "Orbit"



Simulation (center of mass) coordinates



Reduced mass frame; solid black line is position of BH 1 relative to BH 2 (green star); dashed blue line is reference ellipse

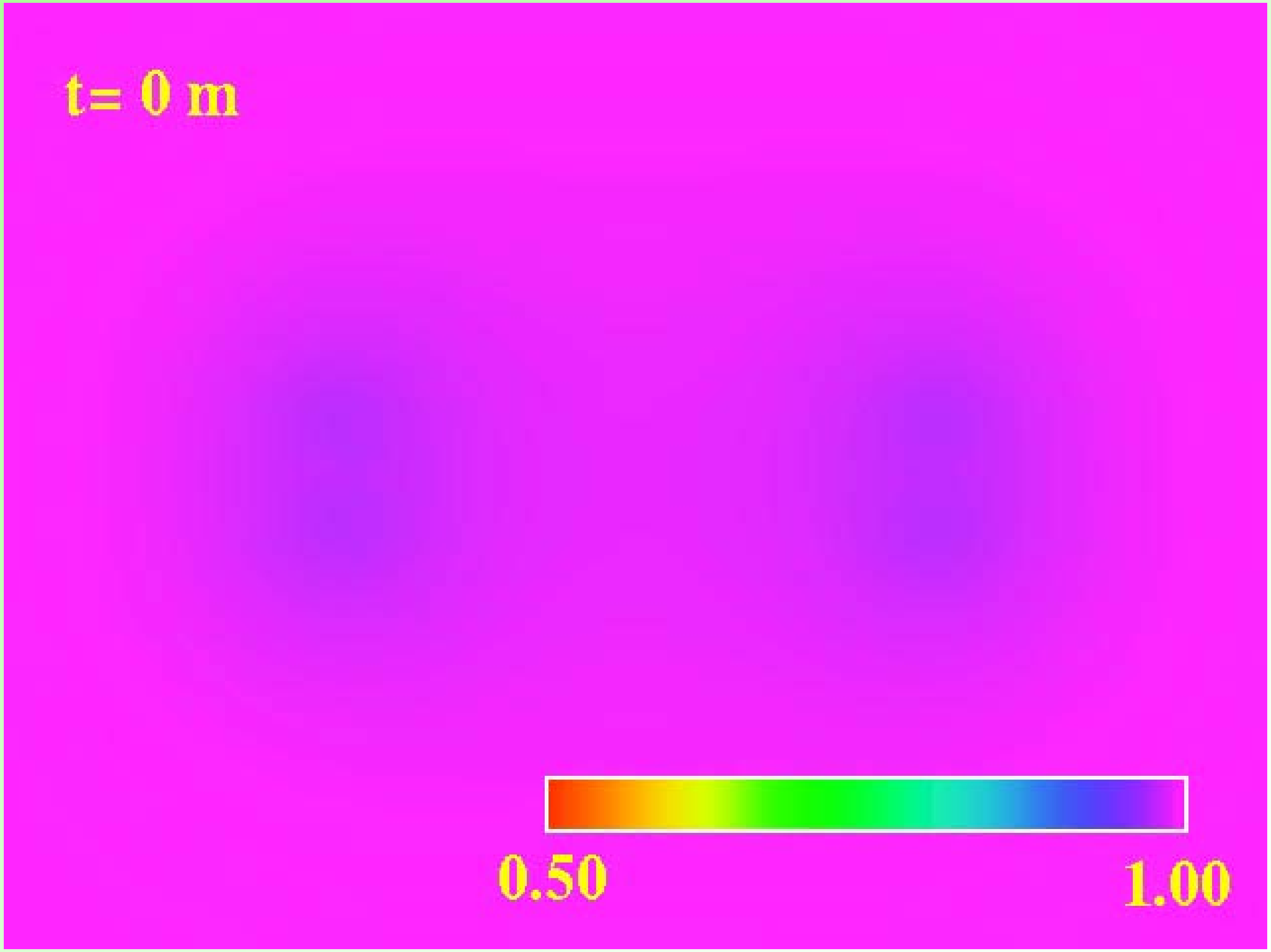
t=0

- Equal mass components
- Eccentricity ~ 0.25
- Coord. Separation ~ 16M
- Proper Separation ~ 20M
- Velocity of each hole ~ 0.12
- Spin ang mom of each hole = 0

t ~ 200

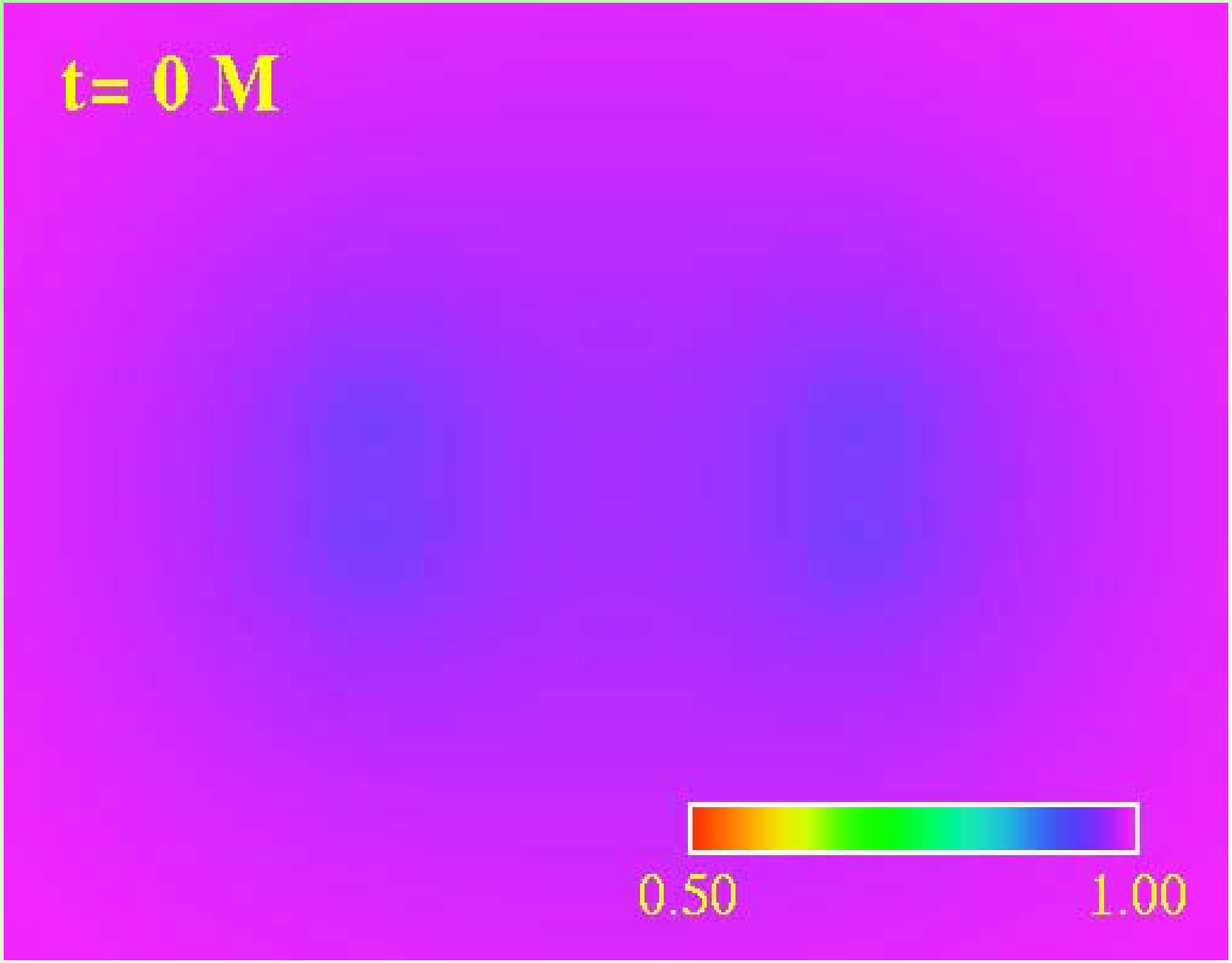
- Final BH mass ~ 1.85M
- Kerr parameter a ~ 0.7
- Estimated error ~ 10%

Case 1: "Lapse function"
(think relativistic gravitational potential)
Uncompactified coordinates



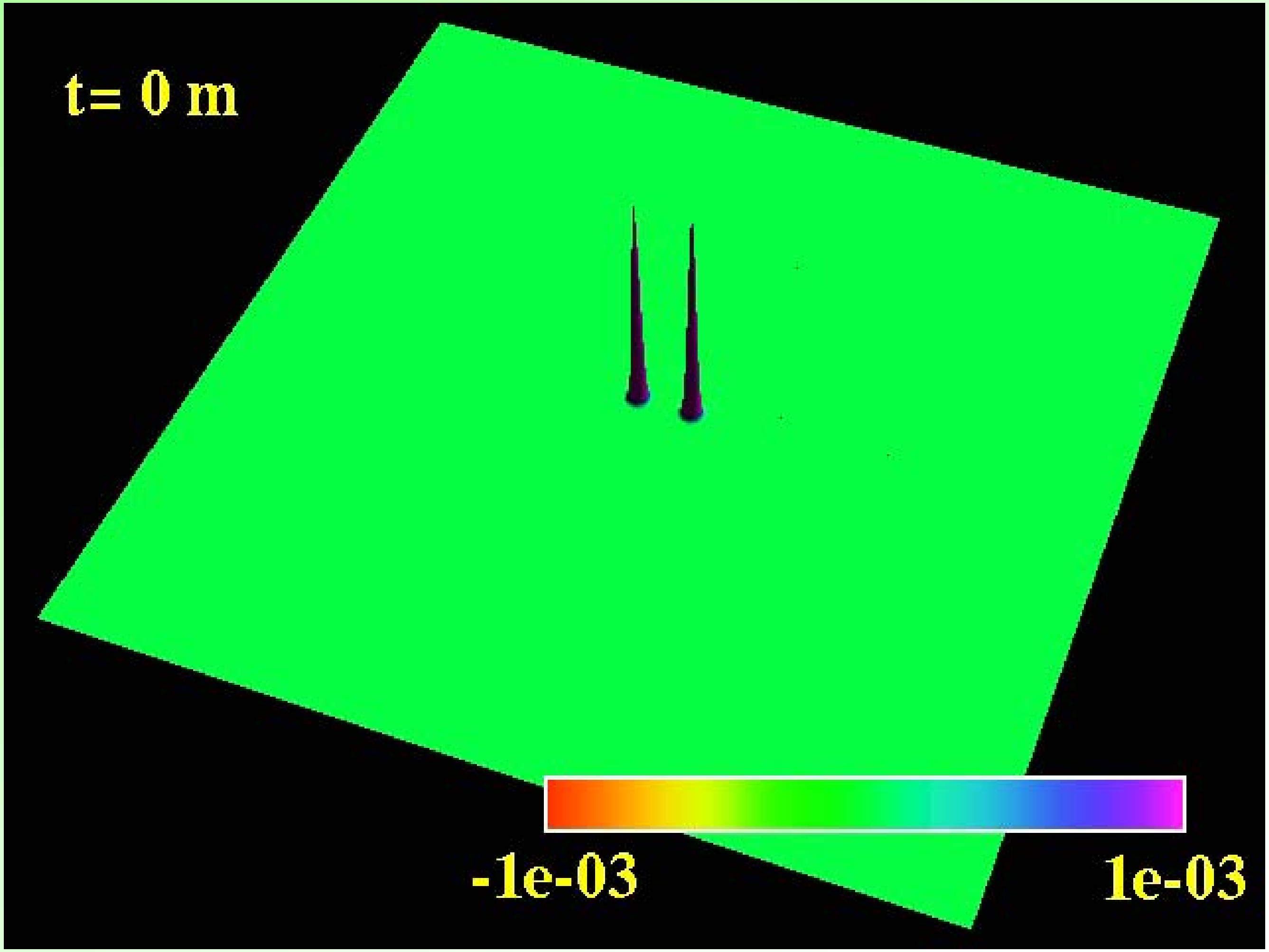
- All animations show quantities on the z=0 plane
- Time measured in units of M

Case 2: "Lapse function"
(pretty much a complete orbit!)



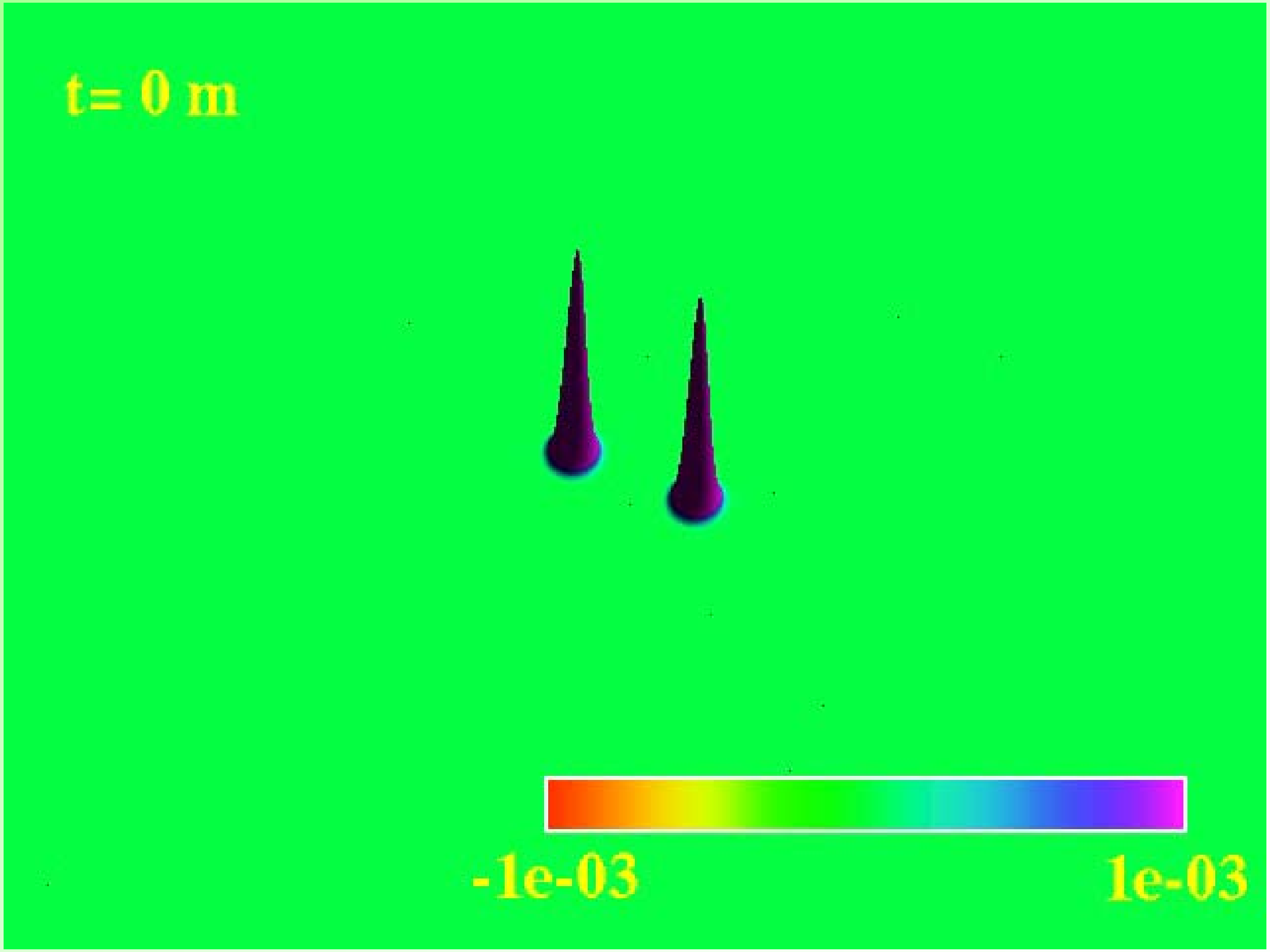
Scalar field modulus

Compactified (code) coordinates

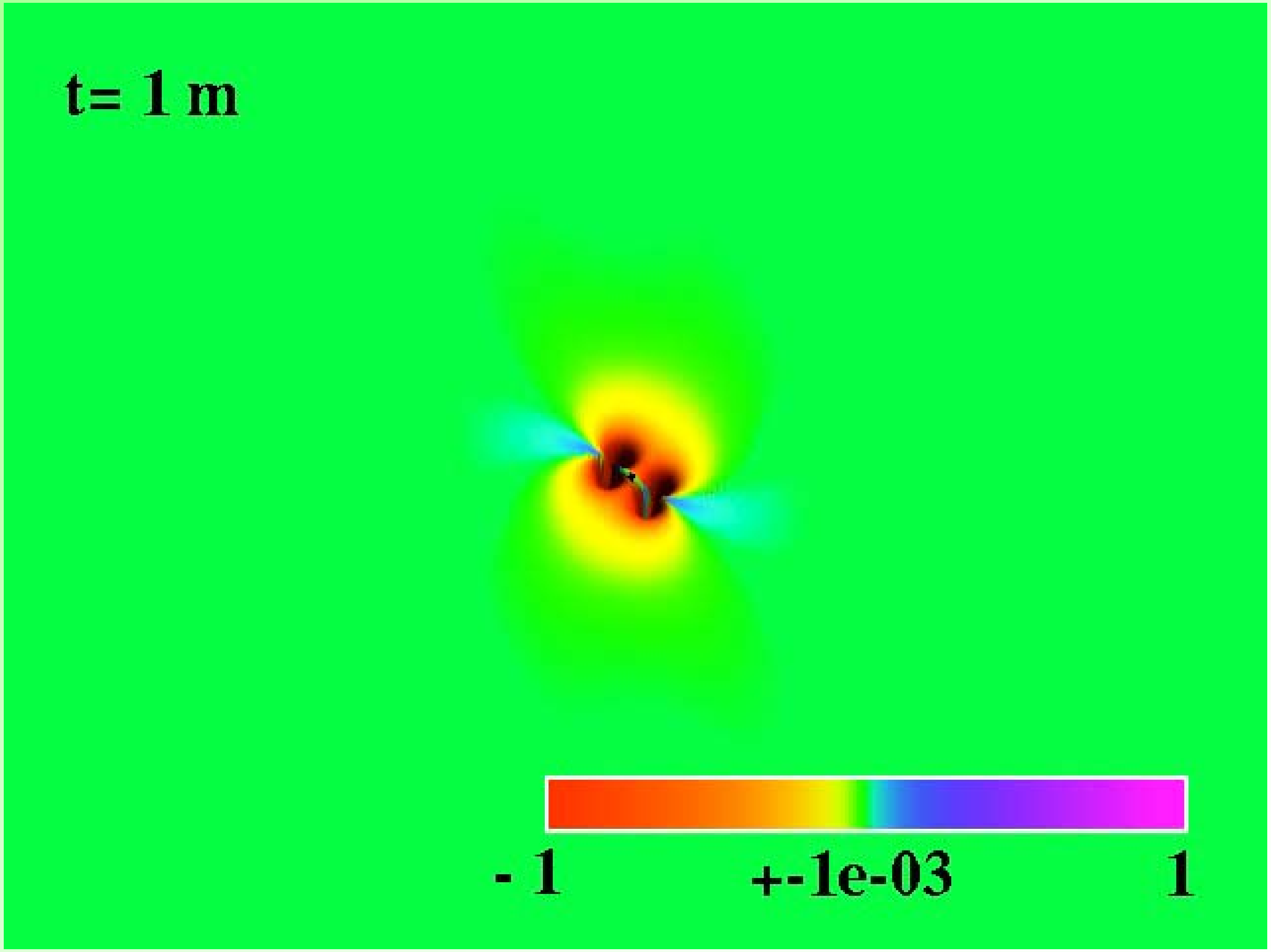


Scalar field modulus

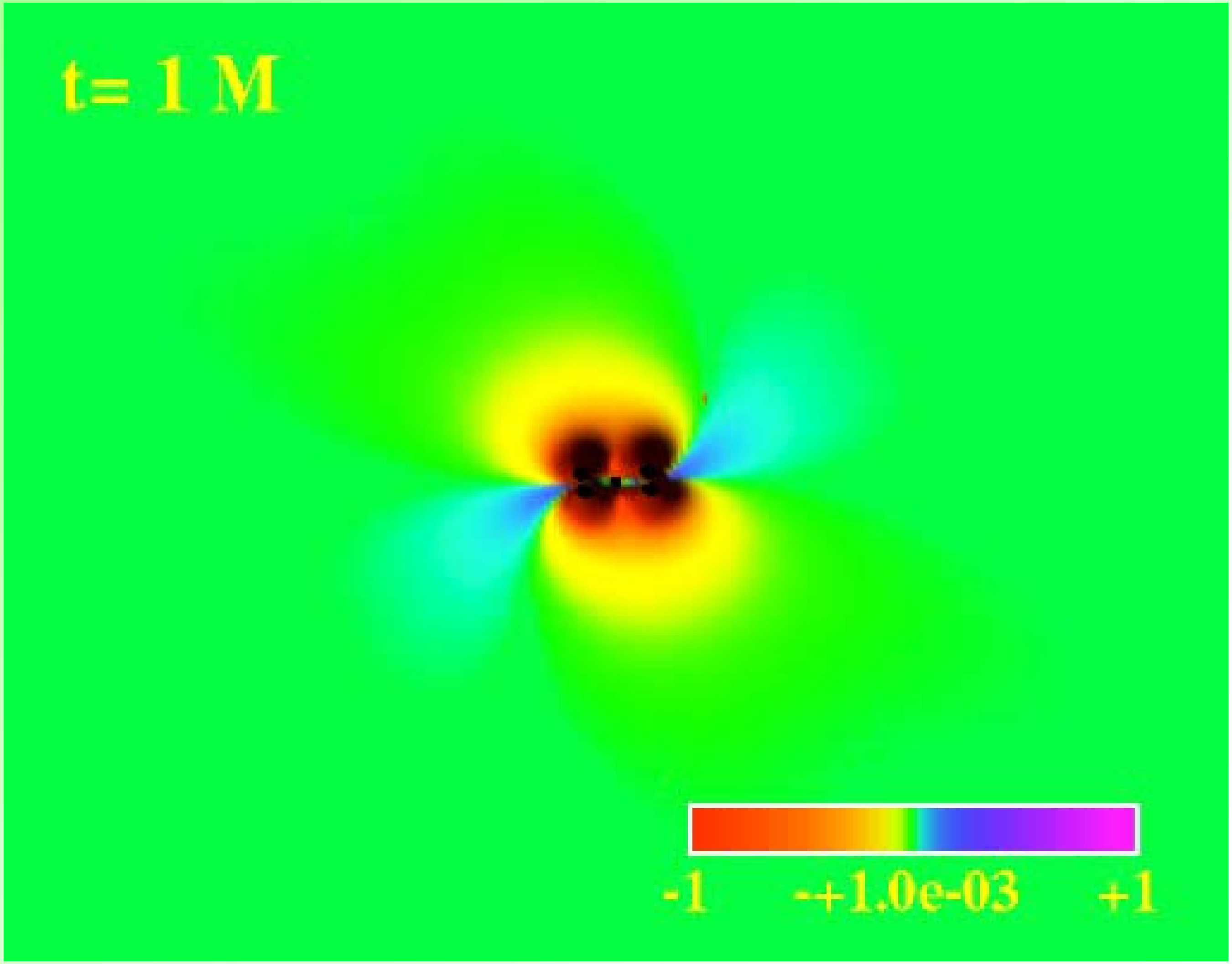
Uncompactified coordinates



Case 1: Gravitational Radiation



Case 2: Gravitational Radiation



Computation vital statistics

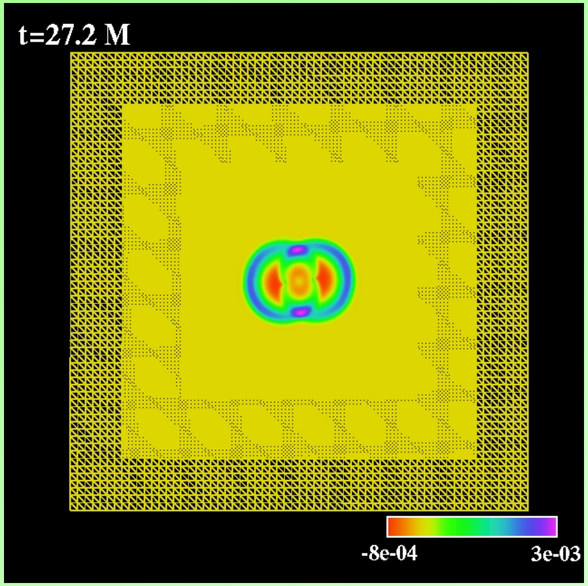
Data shown

- ~ 60,000 time steps on finest level
- CPU time: about 70,000 CPU hours (8 CPU YEARS!)
  - Started on 48 processors of our local P4/Myrinet cluster
  - Continues of 128 nodes of WestGrid P4/gig cluster
- Memory usage: ~ 20 GB total max
- Disk usage: ~ 0.5 TB with infrequent output!!

- Base grid resolution: 48 x 48 x 48
  - 9 levels of 2:1 mesh refinement
    - Effective finest grid 12288 x 12288 x 12288

Sample Mesh Structure

1



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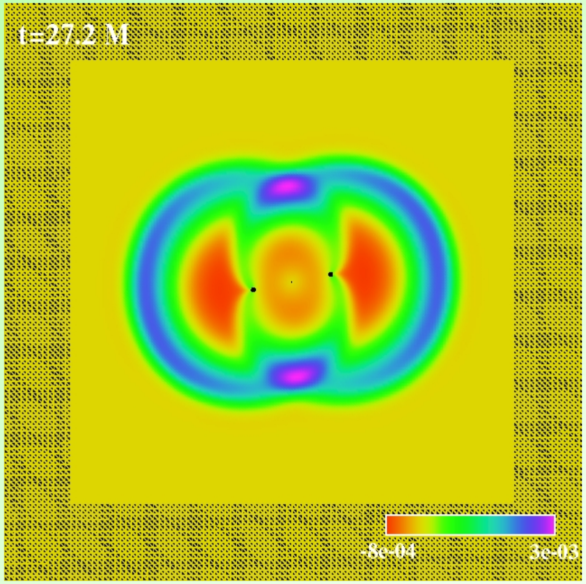

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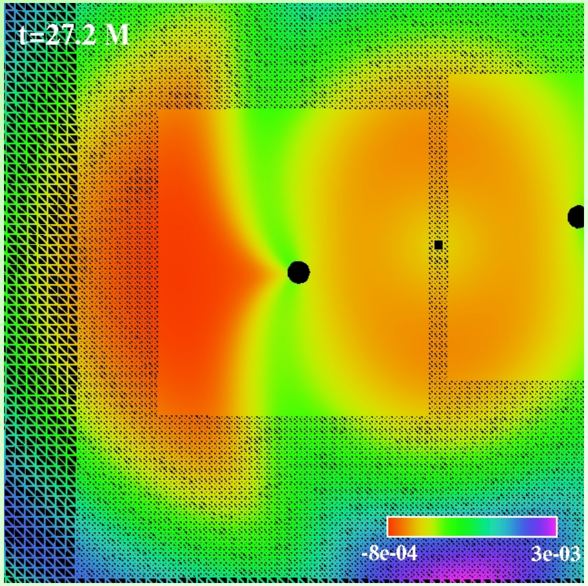

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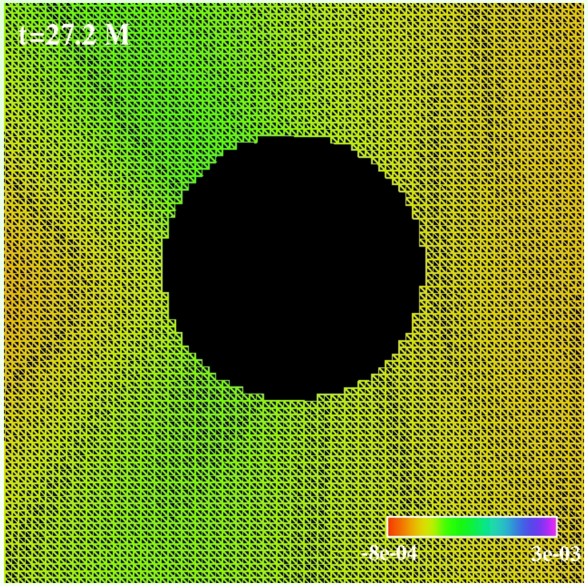
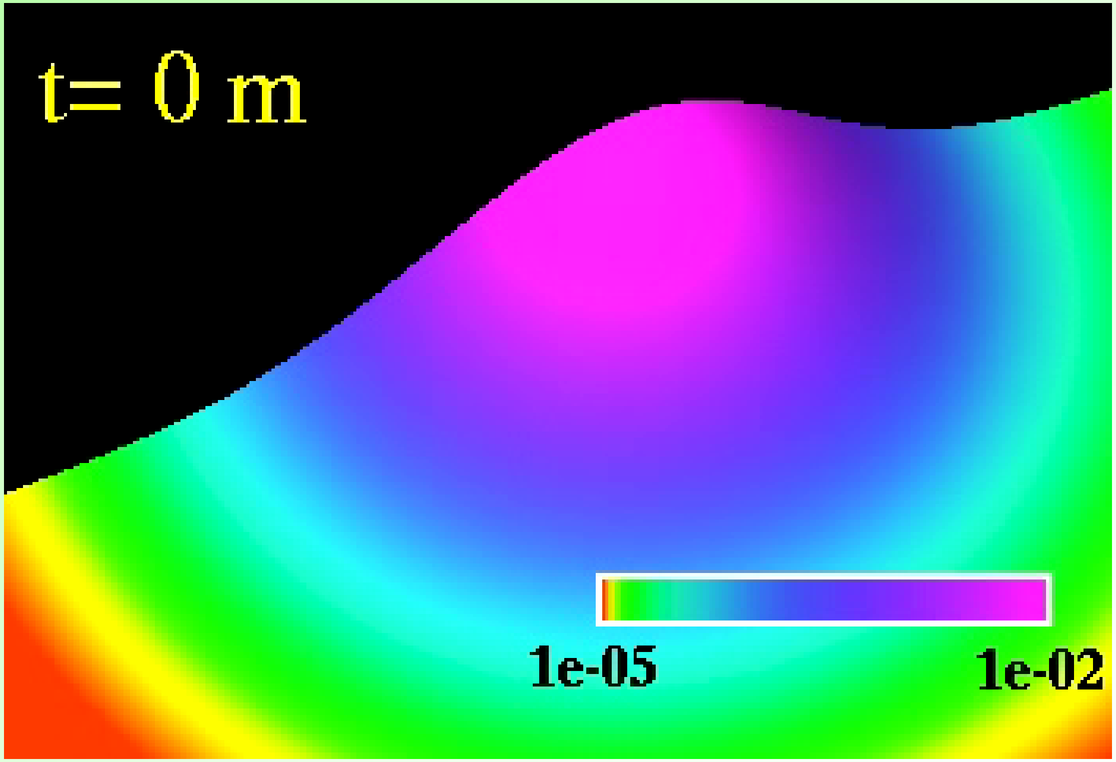


Boson Star – Black Hole Collision: Case 1

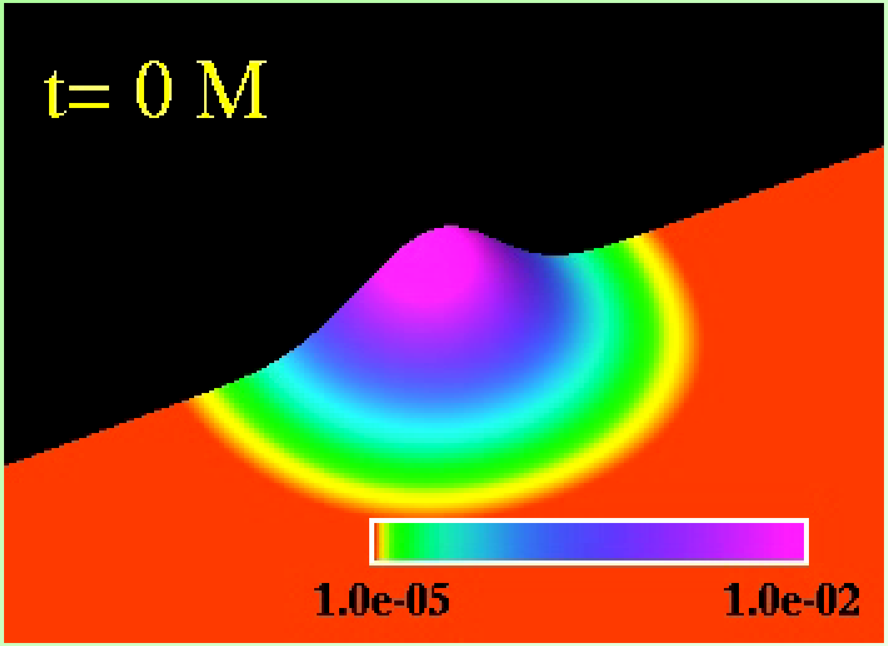
- MBS/MBH ~ 0.75
- RBS/RBH ~ 12.5
- BH initially just outside BH and moving towards it with v ~ 0.1 c

$$\left|\phi(t,\rho,z)\right|$$

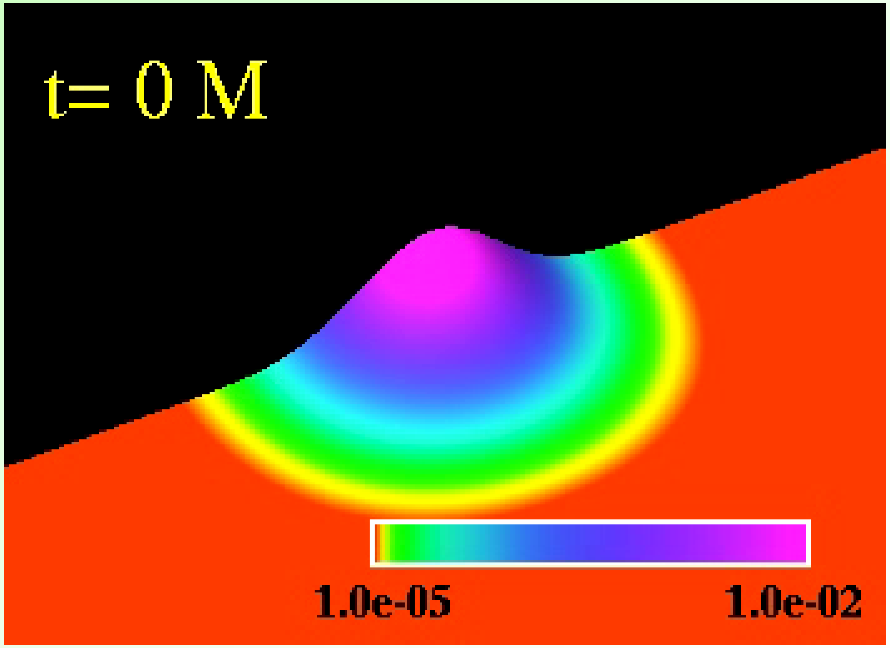


Boson Star – Black Hole Collision: Case 2

- MBS/MBH ~ 3.00
- RBS/RBH ~ 50.0
- BH initially just outside BS, and at rest



mesh spacing 2h



mesh spacing h

Plan of Attack

- Choptuik (4 remaining lectures & labs/tutorials)
  - Basics of numerical analysis, and a strategy for the solution of the time-dependent, non-linear PDEs of mathematical physics, such as fully coupled gravito-hydrodynamics
  - Software tools for above
  - Approaches to Einstein's equations to facilitate generation of NEW solutions
    - Nonlinearity, multi-scale nature crucial here
  - Model problems, gravitational collapse

Big Names in Numerical Analysis

(Read everything by them that you can get your hands on)

- Achi Brandt (multigrid, MLAT, solution of elliptic systems)
- Heinz-Otto Kreiss (solution of time dependent systems)
- Joseph Oliger (solution of time-dep systems & AMR)
- Marsha Berger (AMR & MLAT)
- Randall Leveque (solution of systems of conservation laws [e.g. hydrodynamics], AMR)

Big Names in Numerical Relativity (NR) ?

- CHALLENGE TO CLASS / CLASS EXERCISE:
  - Compile COMPLETE NR biblilography, on-line, by end of this school!
- How does one measure importance?
  - MWC's favorite: Whether there's a new solution of the Einstein equations presented or not

Something to think about

- Of the following metaquestions, which is of most basic importance to the fields of physics and astrophysics, as practiced day-to-day by physicists and astrophysicists
  - HOW?
  - WHAT?
  - WHY?

WHAT!!!

NR as an Empirical Science

- Determining WHAT is fundamentally an EMPIRICAL activity.
- (Un)Fortunately, cannot be learned from lectures; must come from EXPERIENCE in the solution of the Einstein equations
  - Mathematical formulation
  - Discretization (continuum equations -> algebraic equations)
  - Solution of discrete equations
  - Parameter space surveys, analysis and extraction of physics
- Again, aim for general strategies that are likely to be SUFFICIENT for systems such as Einstein equations, hydrodynamics (e.g. LSODA or equivalent for sets of ODEs, second-order Crank-Nicholson for systems of PDEs for fundamental fields, HRSC methods for systems of conservation laws; multigrid for solution of elliptic equations as well as implicit equations from time-dependent schemes; MLAT; AMR; visualization tools; …)
  - Need to become fluent with ALL of the above to succeed in NR

The Numerical Relativist's Mantra

- WHAT DO WE WANT?
  - THE ANSWER!!
- WHEN DO WE WANT IT?
  - NOW!!!

WHAT vs HOW & WHY

- How & Why are seductive, particularly when computers are involved
- Field is currently SHORT on people who routinely advance WHAT, and LONG on people who wish to answer HOW & WHY (about particular schemes applied to the Scwarzschild solution, e.g.)
- Concentrate on What, and keep in mind that, in absence of a solution, there is no need to have THE MOST OPTIMAL ALGORITHM
- Need algorithms with proper scaling (Brandt's Golden Rule), but will, in general be MANY such algorithms
  - Don't worry about constant-factor optimizations until the results are coming in
- Emphasis on techniques and approaches that are SUFFICIENT for success; as (astro)-physicists, we could care less about NECESSARY conditions.

The Cross-Disciplinary Nature of NR

- Physics (Classical Gravitation)
- Astrophysics
- Applied Mathematics
- Numerical Analysis
- Computer Science & Computer Engineering & Computational Science

Time for Discussions (aka COFFEE!!)